Shock Transmission and the Sources of Heterogeneous Expectations*

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Abstract

This paper studies how heterogeneity in expectation formation affects the transmission of macroeconomic shocks. In a general class of macroeconomic models, I first identify a novel channel of shock transmission that works through such heterogeneity. Agents forming expectations observe information about realized variables, and pass it through a model to map from that information to the expectation of interest. I show that shocks transmit through heterogeneous expectations whenever these two components are correlated across agents: when there are systematic relationships between agents' information and subjective models. This has broad implications, as many standard theories of bounded rationality generate such relationships if heterogeneity is permitted in both components of expectations. I then study this effect in a specific application to household beliefs around inflation. Using unique features of a UK survey, I document evidence of my novel channel in this context. In a model matching this data, transmission through expectations heterogeneity is substantial and time-varying. In particular, transitory inflation spikes may become 'baked in' to the expectations of certain households, with persistent effects on future shock transmission.
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1 Introduction

It has been well documented empirically that expectations of macroeconomic variables tend to be extremely heterogeneous. Even within groups of similar agents, expectations of inflation, unemployment, and other variables are extremely dispersed.[1] However, when policymakers study expectations data they typically ignore this, and focus on an average measure of the relevant expectation. Academic work frequently does the same.[2]

Where existing models do feature heterogeneous expectations, the heterogeneity is often a side-effect of underlying frictions, useful for distinguishing between different models or identifying parameters.[3] In this paper I propose an alternative reason to look beyond the first moment of the distribution of expectations: heterogeneous expectations are a *channel* through which shocks transmit to aggregate variables. I show that such effects can be large, and are missed by analysis relying on average expectations alone.

Critically, this transmission channel depends on the underlying *source* of the heterogeneity. To form an expectation an agent takes some information on the realizations of certain variables, and passes it through a model to map from their information to the expectation of interest. In workhorse macroeconomic models, for example, agents observe all variables realized up to the current period (full information), and map from that to expectations using each variable's equilibrium law of motion (rational expectations). Heterogeneity could therefore arise because agents have heterogeneous information, or because they use heterogeneous subjective models to interpret their information - or it could be both.

The first contribution of this paper is to show that in that last case, cross-sectional relationships between information and subjective models generate a novel *narrative heterogeneity* channel of aggregate shock transmission. I characterize this channel using a decomposition of the response of aggregate behavior to shocks in a general log-linear macroeconomic framework, with arbitrary expectation formation. While this is a general result, I then go on to show that the narrative heterogeneity channel has large and time-varying effects in a specific application to household beliefs about inflation. In particular, I find that temporary spikes in inflation may get 'baked in' to the expectations of certain households, with persistent consequences for future shock transmission.

[1] e.g. Carroll (2003), Mankiw et al. (2004), Dovern et al. (2012), Coibion and Gorodnichenko (2012), Andrade and Le Bihan (2013), Ma et al. (2021), Candia et al. (2022).

[2] Recent policy examples include Powell (2022), Mann (2022), Schnabel (2022). In academic work this is necessarily the case in models with a representative agent (e.g. Fuster et al., 2010; Bhandari et al., 2019; Caballero and Simsek, 2022; Gáti, 2022). It is also common when using expectations data in empirical work (e.g. Coibion and Gorodnichenko, 2015; Adam et al., 2022; Doh and Smith, 2022).

[3] e.g. Pfajfar and Santoro (2010); Coibion and Gorodnichenko (2012); Falck et al. (2021); Wang (2022).

The intuition for the narrative heterogeneity channel stems from the fact that an agent receiving possibly noisy information about a variable uses it for two purposes. First, they update expectations about that variable directly, depending on the precision of their information. Second, they update expectations of other variables, depending on how the variables are related in their subjective model of the economy. Information about a given shock therefore causes different reactions in agents with different subjective models. If that information is observed most precisely by agents with particular non-representative models, their subjective model has a disproportionate impact on aggregate expectations, and on aggregate behavior. Formally, the aggregate response to a shock depends on the cross-sectional covariance between the two rounds of updating: between information precision, and the perceived relationships between variables. I refer to this as the narrative heterogeneity channel because a simple definition of a narrative is that it consists of a state of the world (information) and a series of perceived consequences (subjective model) (Gibbons and Prusak, 2020).[4]

On top of this, the decomposition in the first part of the paper contains a second *response heterogeneity* channel. This may imply a further role for heterogeneous expectations, if there is heterogeneity in how agents respond to their own expectations. In this case, a shock is amplified if the resulting changes in expectations are largest among those who respond most strongly to those expectations. This extends the well-known effects of heterogeneity in marginal propensities to consume (Auclert, 2019; Bilbiie, 2019) to expectations. Evidence for such relationships with expectations is provided in Macaulay and Moberly (2022).

However, while examples of the response heterogeneity channel have appeared in some recent literature (Broer et al., 2020; Grimaud, 2021; Nord, 2022), existing theoretical models do not allow for the narrative heterogeneity channel. This is because they only allow heterogeneity in either information or subjective models, but not both.[5] Such approaches require minimal deviations from workhorse models with full information and rational expectations, but are at odds with growing evidence for heterogeneity across both components of expectations in a variety of contexts.[6] Moreover, if this two-sided heterogeneity were permitted, many standard models of information frictions and subjective model formation would imply

[4]I use narratives here to mean stories an agent might use to form expectations, rather than narrative identification as in Romer and Romer (2004). See the related literature section below for how this paper links with other recent models of narratives in economics (e.g. Shiller, 2017; Eliaz and Spiegler, 2020).

[5]See for example Angeletos and Pavan (2009), Broer et al. (2020) for heterogeneous information, and Branch and Evans (2006), Malmendier and Nagel (2016) for heterogeneous subjective models. Models departing from both full information and rational expectations simultaneously (e.g. Angeletos et al., 2020; Bianchi et al., 2021) have so far abstracted from heterogeneity. See also the related literature section below.

[6]See for example Link et al. (2021) for information, Andre et al. (2022b) for subjective models, and Pfajfar and Santoro (2010), Beutel and Weber (2021), and Macaulay and Moberly (2022) for both.

strong systematic relationships between the two components of expectations, suggesting a powerful role for the narrative heterogeneity channel. For example, in models of rational inattention (Maćkowiak et al., 2020), different subjective models imply different incentives to acquire information. And if agents are learning (Evans and McGough, 2020), then observing different information will lead them to form different subjective models. The narrative heterogeneity channel is therefore relevant in a wide range of macroeconomic settings.

Having characterized the narrative heterogeneity channel in this general setting, I then show that it is present and powerful in a specific application, concerning household beliefs about inflation. Empirically, I document that information and subjective models are indeed systematically correlated across households. In a model that accounts for the specific patterns observed, the narrative heterogeneity channel has substantial time-varying effects on the transmission of inflationary shocks. The expectations of a representative agent are not therefore sufficient to understand aggregate dynamics in this context.

To show this, I first use unique features of the Bank of England's Inflation Attitudes Survey to separate information and subjective models about inflation at the household level. Respondents are asked about the information sources they used to arrive at their expectations, and how a hypothetical rise in inflation would affect the strength of the UK economy. The first of these questions concerns information without involving the conclusions drawn from it. The second concerns the respondent's subjective model of how inflation relates to the rest of the economy, without asking about information or expectations.

I document two key patterns in this data. First, households who believe inflation makes little difference to the strength of the economy use less information about inflation than households with other subjective models. Those who believe inflation has positive or negative effects use similar information sources. Crucially, this means there is a systematic relationship between information and subjective models, implying that the narrative heterogeneity channel will operate.

Second, information that inflation is high is associated with more negative subjective models of the effects of inflation, both in the cross-section and over time. A greater proportion of households report that inflation makes the economy weaker in periods with high realized inflation, and within a period, those who believe inflation is currently higher are more likely to hold such negative subjective models. The joint distribution of information and subjective models therefore varies over time, and is systematically related to the state of the economy.

In the final part of the paper I develop a model that is consistent with the empirical results, to evaluate the macroeconomic implications of the narrative heterogeneity channel.

To match the survey data, I impose two conditions on the relationships between information and subjective models across households in this model.

First, I assume that households observe noisy signals about inflation, and that the precision of these signals varies across households to match the first empirical result. That is, households with subjective models in which inflation makes little difference to consumption observe less precise information than those with subjective models in which inflation has larger effects, either positive or negative. Second, to match the remaining empirical results it is necessary to augment this with a belief-updating process, in which households with high perceptions of current inflation update their subjective model towards the view that inflation erodes real incomes. In both the time-series and the cross-section, this implies that high inflation (realized and perceived respectively) is associated with more negative subjective models. While these assumptions are imposed in a reduced-form way to match the survey, I show they can be microfounded using rational inattention (Sims, 2003) and ambiguity aversion (Hansen and Sargent, 2008).

This two-way feedback between information and subjective models has several implications for aggregate dynamics. A selection effect weakens the aggregate effects of information frictions, as the households who precisely observe shocks to inflation are those intending to react strongly to such information. In addition, changes in inflation affect the joint distribution of information and subjective models, generating substantial state-dependent variation in the effects of inflationary shocks. Calibrating the model to the survey data, the narrative heterogeneity channel amplifies the elasticity of aggregate consumption to inflation in steady state by more than 50%, and increases the time-series variation in that transmission by 65%.

Finally, the interaction between the two components of expectations can cause temporarily high inflation to become 'baked in' to the expectations of some households, a concern for many economies in 2022 (Carstens, 2022). Households with subjective models in which inflation strengthens the real economy observe the higher inflation, and update their subjective model to a less positive view. If their long-run expectations also rise, they carry this more neutral model into the following periods, and so pay less attention to inflation going forward. Even if inflation subsequently falls, they do not observe it, and so their expectations remain elevated. Conversely, households with negative subjective models pay more attention, and so observe any disinflation with great precision. The survey evidence is consistent with this mechanism. This selective baking in has persistent effects on the future dynamics of the economy, through a persistent change in the narrative heterogeneity channel. These effects would be missed in an analysis only considering average expectations.

Related literature. This paper principally contributes to the broad literatures on information frictions, subjective models, and heterogeneity in macroeconomics. In recent years, a large literature has documented an important role for heterogeneous household income and wealth in the transmission of macroeconomic shocks (see Kaplan and Violante, 2018, for a review). In particular, Auclert (2019) decomposes the channels of monetary policy transmission, highlighting those operating through heterogeneity in household asset positions. However, as this decomposition is done assuming perfect foresight, it cannot shed light on the heterogeneous macroeconomic expectations studied here.

Heterogeneous expectations are, however, common in models of limited information (see Coibion et al., 2018, for a review). Agents receive idiosyncratic signals (e.g. Sims, 2003), or update information sets in different periods (e.g. Reis, 2006). In addition, incentives to acquire information may differ across agents (Broer et al., 2020; Macaulay, 2021; Ciani et al., 2022). However, these models typically assume that agents know the true equilibrium model of the economy, so they abstract away from the narrative heterogeneity channel. Indeed, in models with no heterogeneity in other agent characteristics, so no response heterogeneity channel, the dispersion in expectations often plays no direct role in shock transmission, and is rather a byproduct of the sluggishness in average expectations that determines aggregate dynamics (e.g. Maćkowiak and Wiederholt, 2009; Coibion and Gorodnichenko, 2015).

Similarly, papers on learning (Evans and McGough, 2020), model uncertainty (Ilut and Schneider, 2022), imperfect common knowledge (Angeletos and Lian, 2018), level-k thinking (Farhi and Werning, 2019), and others study the effects of misperceptions of the true structural relationships in the economy, assuming that agents observe all variable realizations up to the current period (Molavi, 2019). Again, heterogeneous expectations feature frequently in this literature (see Hommes, 2021, for a review), for example because different cohorts use different life experiences to learn about laws of motion (Malmendier and Nagel, 2016). Similarly, heterogeneity in the way investors interpret data (i.e. heterogeneous subjective models) can explain a variety of phenomena in financial markets (Harris and Raviv, 1993; Scheinkman and Xiong, 2003; Banerjee and Kremer, 2010; Atmaz and Basak, 2018; Martin and Papadimitriou, 2022) and labor markets (Jäger et al., 2021; Braun and Figueiredo, 2022). With full information, however, this literature abstracts away from the narrative heterogeneity channel, and in many cases the average subjective model is sufficient to summarize aggregate shock transmission (e.g. Andrade et al., 2019).

Where existing literature does depart simultaneously from both full information and rational expectations, the focus is on settings with a representative agent (Ryngaert, 2018; Bordalo et al., 2018, 2020; Angeletos et al., 2020; Bianchi et al., 2021; Maxted, 2022). However, there is mounting evidence that in many contexts there is substantial heterogeneity in both information (Song and Stern, 2021; Link et al., 2021, 2022) and subjective models (Patton and Timmermann, 2010; Andrade et al., 2019; Laudenbach et al., 2021; Andre et al., 2022b). Pfajfar and Santoro (2010), Madeira and Zafar (2015), Beutel and Weber (2021), and Macaulay and Moberly (2022) find evidence for simultaneous heterogeneity along both dimensions. To my knowledge, this paper is the first to systematically study the transmission effects of simultaneous heterogeneity in these two components of expectation formation.

The empirical part of the paper also contributes to this literature, by separating information from subjective models around inflation in a survey with a long time series. This complements early work on household dislike of inflation (Shiller, 1997), and more recent evidence relating this to expectations of other variables and actions (Kamdar, 2019; Candia et al., 2020). Relatedly, Michelacci and Paciello (2020) and Dräger et al. (2020) document heterogeneity in household preferences over inflation and interest rates, which are plausibly linked to subjective models of how those variables affect other aspects of a household's environment. I extend this by documenting the correlation of those subjective models with household information, which drives the narrative heterogeneity channel.

Finally, while models of narratives have been developed in microeconomics and political economy (Bénabou et al., 2018; Akerlof et al., 2020; Eliaz and Spiegler, 2020), most work in macroeconomics has been concerned with empirically tracking particular narratives and their impacts (Shiller, 2017; Larsen et al., 2021; Goetzmann et al., 2022). Macaulay and Song (2022) in particular find that multiple distinct narratives often circulate about the same economic events: the framework in this paper is a step towards incorporating such heterogeneous narratives into macroeconomic models. Note that the Directed Acyclic Graphs increasingly used to define narratives in this literature (Eliaz and Spiegler, 2020; Andre et al., 2022a; Macaulay and Song, 2022) are nested in the subjective models considered here, as are the prior belief distortions in Flynn and Sastry (2022).

Outline. The rest of the paper is structured as follows. Section 2 derives the novel decomposition of aggregate responses to shocks in a general log-linear model with arbitrary information and subjective models. Section 3 explores information and subjective models about inflation in the data. Section 4 develops a model to match the empirical findings, and Sections 5 and 6 explore the implications of that model. Section 7 concludes.

2 General decomposition

I begin by presenting a decomposition of the effects of an arbitrary shock on the aggregate choices of a group of agents, in a general log-linear model. The decomposition highlights the roles played by information and subjective models, and their distribution across agents, in determining the strength of aggregate shock transmission. The aggregate response to a shock comes through three channels: the representative agent channel, the response heterogeneity channel, and the narrative heterogeneity channel.

2.1 The agent

Agent $i \in I$ chooses a $N_x \times 1$ vector of choice variables $\boldsymbol{X}_t^i$ in period t. Letting lower case letters be log-deviations of variables from some arbitrary point, a log-linear approximation of their policy function can be written:[7]

$$\boldsymbol{x}_t^i = \boldsymbol{\mu}_t^i \mathbb{E}_t^i \boldsymbol{z}_t^i \tag{1}$$

where $\boldsymbol{z}_t^i$ is a $N_z \times 1$ vector of variables taken as given by the agent,[8] and $\boldsymbol{\mu}_t^i$ is a $N_x \times N_z$ matrix of coefficients. This can be thought of as the log-linearized solution to some optimization problem, which has been left in the background.

The vector $\boldsymbol{z}_t^i$ may include both aggregate and idiosyncratic variables. Some elements of $\boldsymbol{z}_t^i$ may be known precisely by the agent; for the unknown elements, the agent-specific expectations operator $\mathbb{E}_t^i$ may or may not coincide with rational expectations. The elements of $\boldsymbol{z}_t^i$ may also be realized in any period: the indexation at time t simply reflects that they are the variables that matter for period t choices. This setup therefore encompasses a wide range of models, for choices made by households, firms, investors, and other types of agent. I show a particular example with a standard household consumption-saving problem in Section 2.2.

I now consider a shock ξ_t, which affects some or all of the variables in $\boldsymbol{z}_t^i$. The reaction of agent choices is determined by the effects of the shock on the expectation of each element

[7]This linearization need not be taken about a steady state, or about the same point for each agent. If two agents have different idiosyncratic state variables, they can therefore have different responses to aggregate variables and expectations, just as they would in a fully non-linear model. This is why the coefficients $\boldsymbol{\mu}_t^i$ are indexed by agent and by period, as the linearization could be taken about different points each period.

[8]This is without loss of generality, as any endogenous choice variable can also be expressed as a linear function of other elements of $\boldsymbol{z}_t^i$. Substituting out using that function, and repeating for any remaining endogenous variables, gives a policy function only in terms of variables exogenous to the agent.

of the policy function:

$$\frac{d\boldsymbol{x}_t^i}{d\xi_t} = \boldsymbol{\mu}_t^i \frac{d\mathbb{E}_t^i \boldsymbol{z}_t^i}{d\xi_t} \tag{2}$$

Applying the chain rule to the derivative of each element of $\mathbb{E}_t^i \boldsymbol{z}_t^i$ leads to a simple expression for the agent's response to the shock.

Proposition 1 *For any agent with policy function described by equation 1, the response to a shock ξ_t is given by:*

$$\frac{d\boldsymbol{x}_t^i}{d\xi_t} = \boldsymbol{\mu}_t^i (\boldsymbol{I} - \boldsymbol{\mathcal{M}}_t^i)^{-1} \boldsymbol{\delta}_t^i \tag{3}$$

where:

$$\boldsymbol{\mathcal{M}}_t^i = \begin{pmatrix} 0 & \mathcal{M}_{12,t}^i & \dots & \mathcal{M}_{1N_z,t}^i \\ \mathcal{M}_{21,t}^i & 0 & \dots & \mathcal{M}_{2N_z,t}^i \\ \vdots & \vdots & \ddots & \vdots \\ \mathcal{M}_{N_z1,t}^i & \mathcal{M}_{N_z2,t}^i & \dots & 0 \end{pmatrix}, \qquad \mathcal{M}_{jk,t}^i \equiv \frac{\partial \mathbb{E}_t^i z_{jt}^i}{\partial \mathbb{E}_t^i z_{kt}^i} \tag{4}$$

$$\boldsymbol{\delta}_t^i = \left(\left.\frac{d\mathbb{E}_t^i z_{1t}^i}{d\xi_t}\right|_{\mathbb{E}_t^i z_{m\neq 1,t}}, \left.\frac{d\mathbb{E}_t^i z_{2t}^i}{d\xi_t}\right|_{\mathbb{E}_t^i z_{m\neq 2,t}}, \dots, \left.\frac{d\mathbb{E}_t^i z_{N_zt}^i}{d\xi_t}\right|_{\mathbb{E}_t^i z_{m\neq N_z,t}} \right)'$$

Proof. Appendix A.1 ■

Equation 3 is useful because it distinctly highlights the separate roles played by the agent's information, subjective model, and policy function coefficients in determining the behavioral response to the shock. When the shock occurs, agent i first receives some direct information about how each of the variables in $\boldsymbol{z}_t^i$ have changed, and updates their expectations of each according to $\boldsymbol{\delta}_t^i$. Importantly, each element of $\boldsymbol{\delta}_t^i$ is defined as the update to that expectation in response to the shock, *holding constant* the expectations of all other variables. This first update does not therefore involve passing information about other variables through the agent's subjective model of how the variables relate to each other. $\boldsymbol{\delta}_t^i$ therefore captures a broad notion of the information observed about each variable, separately from that subjective model.[9] The j^{th} element of $\boldsymbol{\delta}_t^i$ will be zero for an agent who obtains no direct information about the corresponding z_{jt}^i, and will rise to the realized change $dz_{jt}^i/d\xi_t$ with perfect observation. If the agent is Bayesian, then between these extremes $\boldsymbol{\delta}_t^i$ reflects the signal-to-noise ratio of observed information (see the example in Section 2.2). Otherwise,

[9]Note that I am agnostic here about how the agents acquire this information, so this encompasses models of exogenous noisy information (Lucas, 1972), rational inattention (Sims, 2003), information avoidance (Golman et al., 2017), social learning (Mobius and Rosenblat, 2014), and others.

$\boldsymbol{\delta}_t^i$ simply reflects the agent's non-Bayesian use of direct information (e.g. De Filippis et al., 2022).

This, however, does not capture the entire response of expectations to the shock. After updating the expectation of each variable through the direct information effect, the agent engages in a second round of updating, where they use their newly updated expectations of each z_{jt} to inform their expectations of all other variables that they believe to be linked to z_{jt} through their subjective model. This secondary updating is reflected by $(\boldsymbol{I} - \boldsymbol{\mathcal{M}}_t^i)^{-1}$.[10] Once all expectations have been updated, the coefficients $\boldsymbol{\mu}_t^i$ determine the choice response.

Importantly, while the matrix $\boldsymbol{\mathcal{M}}_t^i$ reflects the direct effect of expectations of one variable on another, variables may also be linked indirectly. That is, an update to $\mathbb{E}_t^i z_{jt}^i$ may affect $\mathbb{E}_t^i z_{kt}^i$ directly, but also indirectly through its effect on the expectation of some other variable $\mathbb{E}_t^i z_{lt}^i$, which is linked in the household's subjective model to both z_{jt}^i and z_{kt}^i. The matrix $(\boldsymbol{I} - \boldsymbol{\mathcal{M}}_t^i)^{-1}$ captures all such direct and indirect links between variables. From here, it will be convenient to work directly with this, which I refer to as the cross-learning matrix:[11]

$$\boldsymbol{\chi}_t^i \equiv (\boldsymbol{I} - \boldsymbol{\mathcal{M}}_t^i)^{-1} \tag{5}$$

where the $(j,k)^{th}$ element of $\boldsymbol{\chi}_t^i$ will be denoted $\chi_{jk,t}^i$. It is these values that are measured in the empirical literature on cross-learning (e.g. Roth and Wohlfart, 2020). By allowing for direct and indirect perceived links across variables, this nests a wide range of possible subjective models, including those involving many variables (e.g. Crump et al., 2021).

Finally, having updated all of their expectations using their information, and then again using their subjective model, agent choices are determined by their reaction to each of those expectations, which is contained in the coefficient matrix $\boldsymbol{\mu}_t^i$. The information, subjective model, and response components of the agent's economic narrative are therefore represented by $\boldsymbol{\delta}_t^i$, $\boldsymbol{\chi}_t^i$, and $\boldsymbol{\mu}_t^i$ respectively.

Notice that full information rational expectations is nested in this framework, as the special case in which all variables realized up to period t are observed, and the subjective model coincides with the true model in equilibrium. This therefore differs from models in which narratives are represented by Directed Acyclic Graphs (DAGs) (Spiegler, 2020): while DAGs are also nested in the notion of subjective models in this section, most general equilibrium

[10] Since the variables held constant in the definition of $\boldsymbol{\delta}_t^i$ may include the actions of other agents, the effects of higher-order beliefs on the perceived optimal use of information also enter through this term.

[11] This has a parallel in the literature on production networks (Carvalho and Tahbaz-Salehi, 2019). The direct links in $\boldsymbol{\mathcal{M}}_t^i$ are analogous to the elements of the input-output matrix, and $\boldsymbol{\chi}_t^i$ is the corresponding Leontief inverse. As with production networks, this Leontief inverse regulates the transmission of shocks.

models do not have a recursive causal ordering of variables, so the true equilibrium laws of motion cannot be expressed as a DAG.

2.2 An example

Consider the textbook setup where infinitely lived households have CRRA utility over consumption, and can trade one-period risk-free bonds for intertemporal consumption smoothing. The consumption function of household i log-linearized about steady state is:

$$c_t^i = (1-\beta)\sum_{s=0}^{\infty}\beta^s \mathbb{E}_t^i y_{t+s} - \sigma\beta\sum_{s=0}^{\infty}\beta^s(\mathbb{E}_t^i r_{t+s} - \mathbb{E}_t^i \pi_{t+s+1}) \tag{6}$$

where y_t is real income in period t, r_t is the nominal interest rate, and π_t is gross inflation. The parameters β and σ are the discount factor and coefficient of relative risk aversion respectively. See Appendix A.2 for the derivation.

This is the familiar result that consumption depends on the expected present value of future income and all expected future real interest rates. Within the framework of equation 1, $\boldsymbol{z}_t^i$ contains all current and future realizations of y_t, r_t, and π_{t+1}. The coefficients $\boldsymbol{\mu}_t^i$ contain the relevant combinations of the preference parameters β and σ.

To see the interpretation of Proposition 1 in more detail, assume that households believe inflation and income are linked according to a simple subjective model:

$$\begin{aligned} y_t &= \alpha^i \pi_t + u_{yt}, \quad u_{yt} \sim N(0, \sigma_y^2) \\ \pi_t &= u_{\pi t}, \quad u_{\pi t} \sim N(0, \sigma_\pi^2) \end{aligned} \tag{7}$$

That is, inflation may have causal effects on real incomes, but there is believed to be no feedback from real incomes to inflation. For this example, assume that the household does not believe r_t is related to either y_t or π_t, so we can leave that out of the analysis.

The household observes a noisy signal about each variable of interest in period t:

$$\begin{aligned} s_{yt}^i &= y_t + \varepsilon_{yt}^i, \quad \varepsilon_{yt}^i \sim N(0, \sigma_{\varepsilon y}^2) \\ s_{\pi t}^i &= \pi_t + \varepsilon_{\pi t}^i, \quad \varepsilon_{\pi t}^i \sim N(0, \sigma_{\varepsilon \pi}^2) \end{aligned} \tag{8}$$

If the household follows Bayes' rule to incorporate these signals into their expectations of y_t and π_t, their posterior expectations of each are a linear combination of s_{yt}^i and $s_{\pi t}^i$, with the weights depending on the relative signal-to-noise ratios of each signal. Importantly, those ratios depend on α^i, as that determines how strongly the variables are believed to be linked, and therefore how informative s^i_{yt} is about π_t, and similarly how informative $s^i_{\pi t}$ is about y_t. Rearranging the resulting expressions for posterior expectations gives:

$$\begin{aligned}\mathbb{E}^i_t y_t &= \frac{\sigma^2_y}{\sigma^2_y+\sigma^2_{\varepsilon y}} s^i_{yt} + \alpha^i \frac{\sigma^2_{\varepsilon y}}{\sigma^2_y+\sigma^2_{\varepsilon y}} \mathbb{E}^i_t \pi_t \\ \mathbb{E}^i_t \pi_t &= \frac{\sigma^2_\pi}{\sigma^2_\pi + \sigma^2_{\varepsilon\pi}\left(1+\alpha^{i2}\frac{\sigma^2_y}{\sigma^2_\pi}\right)} s^i_{\pi t} + \alpha^i \frac{\sigma^2_{\varepsilon\pi}\frac{\sigma^2_y}{\sigma^2_\pi}}{\sigma^2_\pi + \sigma^2_{\varepsilon\pi}\left(1+\alpha^{i2}\frac{\sigma^2_y}{\sigma^2_\pi}\right)} \mathbb{E}^i_t y_t\end{aligned} \tag{9}$$

After a shock ξ_t that moves both y_t and π_t, these expectations change according to:

$$\begin{aligned}\frac{d\mathbb{E}^i_t y_t}{d\xi_t} &= \frac{\sigma^2_y}{\sigma^2_y+\sigma^2_{\varepsilon y}} \frac{dy_t}{d\xi_t} + \alpha^i \frac{\sigma^2_{\varepsilon y}}{\sigma^2_y+\sigma^2_{\varepsilon y}} \frac{d\mathbb{E}^i_t \pi_t}{d\xi_t} \\ \frac{d\mathbb{E}^i_t \pi_t}{d\xi_t} &= \frac{\sigma^2_\pi}{\sigma^2_\pi + \sigma^2_{\varepsilon\pi}\left(1+\alpha^{i2}\frac{\sigma^2_y}{\sigma^2_\pi}\right)} \frac{d\pi_t}{d\xi_t} + \alpha^i \frac{\sigma^2_{\varepsilon\pi}\frac{\sigma^2_y}{\sigma^2_\pi}}{\sigma^2_\pi + \sigma^2_{\varepsilon\pi}\left(1+\alpha^{i2}\frac{\sigma^2_y}{\sigma^2_\pi}\right)} \frac{d\mathbb{E}^i_t y_t}{d\xi_t}\end{aligned} \tag{10}$$

Combining these two equations to solve for each expectation change yields the form in equation 3. The first terms of each equation contain the elements of $\boldsymbol{\delta}^i_t$, and the coefficients in the second terms contain the elements of $\boldsymbol{\mathcal{M}}^i_t$.[12]

Consider first the change in $\mathbb{E}^i_t y_t$. The first term has two components: the signal-to-noise ratio of the income signal s^i_{yt}, and the underlying response of y_t to the shock. That is, if they precisely observe y_t, then $\mathbb{E}^i_t y_t$ responds to the shock in exactly the same way as the realized variable, regardless of changes in $\mathbb{E}^i_t \pi_t$. The noisier the household's direct information about y_t, the smaller that direct response. At the extreme with no direct information observed about y_t ($\sigma^2_{\varepsilon y} \to \infty$), the direct effect of the shock on expectations approaches 0 and the only way the household can update $\mathbb{E}^i_t y_t$ is through $\mathbb{E}^i_t \pi_t$.

The coefficient in the second term also has two components. First, a change in expected inflation only affects expected income if the household believes that the two are linked in their subjective model ($\alpha^i \neq 0$). The slope of the perceived relationship between them, α^i, therefore regulates the updating from $\mathbb{E}^i_t \pi_t$ to $\mathbb{E}^i_t y_t$. Second, this slope from the subjective model is scaled by a factor equal to one minus the signal-to-noise ratio. Intuitively, this scaling reflects how strongly the household weights the information in $\mathbb{E}^i_t \pi_t$ relative to the other information they have about y_t.

Now turn to the change in $\mathbb{E}^i_t \pi_t$. All of the effects described above are present, but there

[12]The equations have precisely the form of equation 63 used in the proof of Proposition 1.

is a further nuance. The weights on $d\pi_t/d\xi_t$ and $d\mathbb{E}_t^i y_t/d\xi_t$ are no longer determined by the simple signal-to-noise ratio in the relevant direct signal. This is because the first term of the $\mathbb{E}_t^i \pi_t$ updating equation reflects the extent of updating due to $s_{\pi t}^i$, holding $\mathbb{E}_t^i y_t$ constant. Since in the household's subjective model π_t is a direct cause of y_t, this conditioning involves assuming the structural shock u_{yt} offsets the perceived rise in π_t, effectively reducing the informativeness of $s_{\pi t}^i$ when it is used in this way. This distortion is smaller if income shocks are believed to be more volatile relative to inflation shocks, as then y_t is less strongly correlated with π_t in the subjective model.

The core insights, however, remain the same: the direct response varies between 0 (if $\sigma_{\varepsilon\pi}^2 \to \infty$) and the realized change in inflation (if $\sigma_{\varepsilon\pi}^2 = 0$), and the coefficient on $d\mathbb{E}_t^i y_t/d\xi_t$ is determined by the association between π_t and y_t in the subjective model (α^i), and how the household weights that information relative to the direct information.

2.3 Aggregate behavior

I now return to the general case. Consider a unit mass of the agents modeled in Section 2.1. Aggregate choices for each choice variable x_{st}^i are given by:

$$\bar{x}_{st} = \int_0^1 \omega_{st}^i x_{st}^i di \tag{11}$$

where ω_{st}^i denotes a weighting applied to agent i's choice x_{st}^i, such that:

$$\bar{x}_{st} = \mathbb{E}_I x_{st}^i \tag{12}$$

where $\mathbb{E}_I$ denotes the expected value across agents.

Again, consider a shock ξ_t that affects some or all of the variables in agent choice functions. Proposition 1 and the properties of covariances lead us to the following decomposition of the aggregate choice response:

Proposition 2 *The response of aggregate choice $\bar{x}_{st}$ to a shock ξ_t is given by:*

$$\frac{d\bar{x}_{st}}{d\xi_t} = \sum_{j=1}^{N_z}\sum_{k=1}^{N_z}\left[\bar{\mu}_{sj,t}\bar{\chi}_{jk,t}\bar{\delta}_{k,t} + Cov_I(\mu_{sj,t}^i, \chi_{jk,t}^i\delta_{k,t}^i) + \bar{\mu}_{sj,t}Cov_I(\chi_{jk,t}^i, \delta_{k,t}^i)\right] \tag{13}$$

where $\delta_{k,t}^i$ and $\mu_{sj,t}^i$ denote the k^{th} element of $\boldsymbol{\delta}_t^i$ and the $(s,j)^{th}$ element of $\boldsymbol{\mu}_t^i$ respectively, $\bar{\delta}_{k,t}$ and $\bar{\mu}_{sj,t}$ are their aggregate counterparts, and $\bar{\chi}_{jk,t}$ is the aggregate value of $\chi_{jk,t}^i$ across *agents i.*

Proof. Appendix A.1 ■

This decomposition shows that three groups of channels determine the aggregate response to shocks. The first term is the *representative agent channel*: the effects of the average coefficients, subjective model, and information about each variable. This summarizes all shock transmission channels in models with a representative agent, and indeed in many models with heterogeneity, where average expectation formation is sufficient to capture shock responses to first order. In Maćkowiak and Wiederholt (2009), for example, firms acquire idiosyncratic signals about aggregate shocks, but the dynamics of the price level are determined by the average level of inattention. Similarly, in Andrade et al. (2019) households differ in their interpretation of forward guidance announcements, but the aggregate effects of an announcement depend only on the average over this mix of beliefs. Heterogeneity might affect the average information or subjective model sustained in equilibrium, but unless one of the other two terms in the decomposition is non-zero, those averages alone drive aggregate shock transmission.

The second term is the *response heterogeneity channel*. Since $\boldsymbol{\chi}_t^i \boldsymbol{\delta}_t^i$ gives the total expectation response to the shock ($d\mathbb{E}_t^i \boldsymbol{z}_t^i / d\xi_t$), this reflects that shocks will be amplified if the agents whose expectations react the most to the shock are the agents whose actions are most sensitive to those expectations. Macaulay and Moberly (2022) provide evidence of one such correlation, between the behavior of inflation expectations and liquidity constraints among German households. This channel, for other expectations, is also behind the novel dynamics in Broer et al. (2020) and Macaulay (2021). In models with full information and rational expectations, the only way the expectations updates can be heterogeneous across agents is if they are differentially exposed to the shock. In that case the true response of idiosyncratic variables will differ across agents, and so observations of e.g. income will respond in heterogeneous ways to the shock. In this way the response heterogeneity channel nests the transmission effects of correlations between heterogeneous MPCs and shock exposure studied extensively in the heterogeneous-agent literature (Auclert, 2019; Bilbiie, 2019).

Finally, the third term is the *narrative heterogeneity channel*. Heterogeneous expectations can generate a channel of aggregate shock transmission even if every agent has the same policy function, if information ($\delta_{k,t}^i$) is correlated with subjective models ($\chi_{jk,t}^i$) across agents. Subjective models determine an agent's response to a given piece of information, so if information is concentrated among agents with particular non-representative subjective models, that distorts the aggregate response away from the representative agent effect.

This channel is novel to this paper. However, standard theories of information acquisition and subjective model formation will generate such correlations between information and subjective models, if heterogeneity is permitted across both. In models of rational inattention (Maćkowiak et al., 2020), agents with different subjective models will have different incentives to acquire information, leading to systematic relationships between the two. And different observed information will lead to agents forming different subjective models in, for example, models with recursive learning (Evans and McGough, 2020). Existing papers in these literatures only miss the narrative heterogeneity channel because they restrict heterogeneity to *either* information, *or* subjective models, but not both; the relevant covariance is always therefore forced to be zero.[13]

To further highlight the intuition for these channels, consider again the textbook consumption function in equation 6, and a shock that increases future inflation π_{t+1}. If all households believe that higher inflation is associated with lower real incomes, then the average $\chi^i_{y\pi,t}$ is negative, and the aggregate consumption response to the future inflation will be negative. This is the *representative agent channel*. If, however, this pessimistic subjective model of the effects of inflation only takes hold among hand-to-mouth households, then aggregate consumption will respond much more positively to the shock than the average would suggest, because the households who reduce their expected future real incomes are the ones who react the least to their expectations. This is the *response heterogeneity channel*. Finally, if all households are unconstrained, but the pessimistic model of inflation is concentrated among households who do not obtain any information about future inflation, then this again raises the aggregate consumption response. Those households who would update expected future incomes down and reduce consumption if they learned that inflation was about to rise are precisely the households who do not observe the shock, and so do not learn of the shock. This is the *narrative heterogeneity channel*.

It is important to be clear that this is a decomposition, not a solution for aggregate actions. $\boldsymbol{\delta}^i_t$ captures direct information received by agent i, but the information received depends on the true reaction of $\boldsymbol{z}^i_t$ to the shock, which I have taken as given so far. In Section 2.4 I extend this to general equilibrium, where the realized changes in $\boldsymbol{z}^i_t$ may depend on aggregate choices made by agents. Proposition 2, however, remains the main result of this section, as it gives the clearest expression of the channels through which heterogeneous expectation formation affects aggregate shock transmission. The general equilibrium effects explored below serve only to further amplify or dampen these existing channels.

[13] An exception is Berardi (2007), who studies a model where agents learn from heterogeneous information sets. However, he studies equilibrium convergence, and not aggregate shock transmission.

2.4 General Equilibrium

To extend this framework to general equilibrium, I make two further assumptions. I therefore lose some of the generality of Sections 2.1-2.3, while still nesting a range of common models.

Assumption 1 All elements of $\boldsymbol{z}_t^i$ are equal across agents i, and are such that:

$$A\boldsymbol{z}_t + B\bar{\boldsymbol{x}}_t + C\boldsymbol{\xi}_t = 0 \tag{14}$$

where $\boldsymbol{z}_t$ is the common value of $\boldsymbol{z}_t^i$, $\bar{\boldsymbol{x}}_t$ is the vector of aggregate agent choices $\bar{x}_{st}$, and $\boldsymbol{\xi}_t$ is a $N_\xi \times 1$ vector of exogenous shocks. A, B, C are coefficient matrices, with dimensions $N_z \times N_z$, $N_z \times N_x$, and $N_z \times N_\xi$ respectively.

As with the choice function (equation 1), equation 14 can be thought of as a log-linearization of N_z structural equations, in this case general equilibrium consistency requirements derived from resource constraints and/or the optimization of other agents beyond those choosing $\boldsymbol{x}_t^i$. For example, if $\bar{\boldsymbol{x}}_t$ are the aggregate choices of households, then equation 14 may contain conditions derived from firm optimization (e.g. a Phillips curve), policy rules, and market clearing conditions. Note that this may require extending the set of variables included in $\boldsymbol{z}_t$, if there are aggregate variables involved in the general equilibrium conditions which do not enter the choice functions for $\boldsymbol{x}_t^i$. For clarity I continue to refer to the agents choosing $\boldsymbol{x}_t^i$ as 'the agents' here.

The key restriction this places on the framework introduced in Sections 2.1-2.3 is that $\boldsymbol{z}_t$ may no longer contain idiosyncratic variables. However, with appropriate redefinitions of variables, this restriction is mild. For example, idiosyncratic income variation could be incorporated by specifying that the incomes of different groups of households are each included as separate variables within $\boldsymbol{z}_t$. Households from one group would simply have zeroes in their $\boldsymbol{\mu}_t^i$ coefficient matrices corresponding to the incomes of groups other than their own.

With this I define a *temporary equilibrium* (Grandmont, 1977; Woodford, 2013), which takes the agents' expectations as given, and defines all other variables such that, conditional on those expectations, agent decisions follow their choice functions and all general equilibrium conditions are satisfied.

Equilibrium definition. Given an exogenous shock vector $\boldsymbol{\xi}_t$ and agent expectations $\mathbb{E}_t^i\boldsymbol{z}_t$, a temporary equilibrium consists of values for aggregate variables $[\bar{\boldsymbol{x}}_t, \boldsymbol{z}_t]$ such that:

1. *Agents:* agents choose $\boldsymbol{x}_t$ according to their choice function (equation 1).
2. *Other Variables:* $\boldsymbol{z}_t$ is such that all general equilibrium conditions are satisfied (equation 14).

Note that for any process of expectation formation, existence of the temporary equilibrium is a necessary condition for existence of the full equilibrium in which expectations are formed endogenously. I restrict attention here to cases in which equilibrium exists, and is continuous in all elements of $\boldsymbol{\xi}_t$.

So far, agent expectations have been formed using general processes. To tractably solve for choice responses in general equilibrium, I now restrict the form of the information component of those expectations.

Assumption 2 Agent information is such that:

$$\boldsymbol{\delta}_t^i = \tilde{\boldsymbol{\delta}}_t^i \frac{d\boldsymbol{z}_t}{d\xi_t} \tag{15}$$

where ξ_t is an element of $\boldsymbol{\xi}_t$, and $\tilde{\boldsymbol{\delta}}_t^i$ is independent of $\boldsymbol{z}_t$.

That is, the direct update to expectations of each element of $\boldsymbol{z}_t$, given a shock ξ_t, is proportional to the realized change in that variable. $\tilde{\boldsymbol{\delta}}_t^i$ therefore reflects the strength of the direct updating of expectations through information, *relative* to the update that would be seen under full information. While this form does not cover all possible information structures, it is consistent with e.g. Bayesian updating under Gaussian uncertainty (see the example in Section 2.2).

With these two assumptions, the response of aggregate choices to a shock ξ_t, inclusive of general equilibrium effects, is given by Proposition 3.

Proposition 3 *Under Assumptions 1 and 2, the general equilibrium response of $\bar{\boldsymbol{x}}_t$ to a shock ξ_t is given by:*

$$\frac{d\bar{\boldsymbol{x}}_t}{d\xi_t} = -\mathbb{E}_I\left(\boldsymbol{\mu}_t^i \boldsymbol{\chi}_t^i \tilde{\boldsymbol{\delta}}_t^i\right)\left(A + B\mathbb{E}_I\left(\boldsymbol{\mu}_t^i \boldsymbol{\chi}_t^i \tilde{\boldsymbol{\delta}}_t^i\right)\right)^{-1} C e_\xi \tag{16}$$

where e_ξ is a $N_\xi \times 1$ vector with zero in every element, except for 1 in the element corresponding to the shocked element of $\boldsymbol{\xi}_t$.

Proof. Appendix A.1. ■

If $B = 0$, there is no general equilibrium feedback from agent choices to $\boldsymbol{z}_t$. This then reduces to the partial equilibrium result in Proposition 2. With $B \neq 0$ there are general equilibrium channels that affect agent choices. However, both the initial partial equilibrium response and the resulting feedback are determined by the product $\mathbb{E}_I(\boldsymbol{\mu}_t^i \boldsymbol{\chi}_t^i \tilde{\boldsymbol{\delta}}_t^i)$, which can still be decomposed into the three channels in Proposition 2.

This is not surprising: if the narrative heterogeneity channel amplifies the partial equilibrium response of agent choices to a particular shock, then it will also amplify the responses of other variables that depend on those choices. If those variables feed back into choices in general equilibrium, that will either amplify or dampen the choice response, depending on the role of that variable in agent choice functions. Whatever the form of this general equilibrium effect, it is still driven by the initial partial equilibrium channels.

In the remainder of the paper I go on to study the narrative heterogeneity channel in the particular case of household beliefs about inflation. The results in this section, however, are more general. In any situation with heterogeneity in how agents form expectations, understanding aggregate dynamics requires understanding the three channels presented in Proposition 2.

3 Survey evidence on information and subjective models of inflation

In this section I take the narrative heterogeneity channel to data, documenting three empirical results about the information and subjective models used by households. Specifically, the results refer to the information UK households obtain about inflation, and their subjective models of how inflation is related to aggregate economic performance. These results indicate the presence of a narrative heterogeneity channel, which varies over time. They will be used to inform the model in Section 4.

3.1 Data

To study the joint behavior of information and subjective models, we need data that is informative about each separately. This is a challenge, as most empirical papers on information frictions or subjective models use data on realized expectations, which combine both information and subjective models (as shown in Section 2), and so cannot be used to identify the narrative heterogeneity channel. I use data from the Bank of England Inflation Attitudes

Survey (IAS), which contains several unique questions which enable me to measure these components of expectation formation separately.

The IAS is a quarterly survey of a repeated cross-section of UK households, run since 2001 (annual until 2003). After weighting, the sample is representative of the UK adult population. I use the individual-level response data from 2001-2019, omitting the quarters conducted after the onset of the Covid-19 pandemic, as the implementation of the survey had to be changed substantially at this time (see Bank of England, 2020).

Alongside questions on expectations of inflation, interest rates, and other macroeconomic and personal variables, respondents are asked several questions which do not commonly appear in other household surveys. These questions are helpful in disentangling information and subjective models about inflation.

The first of these asks households about their subjective model of the relationship between inflation and the 'strength of the economy'.

Question 1 *If prices started to rise faster than they are now, do you think Britain's economy would end up stronger, or weaker, or would it make little difference?*

This differs from standard questions on expected future economic outcomes because it does not invoke the use of information about the state of the world. Similarly to the hypothetical vignettes used in Andre et al. (2022b), the answers to this question are informative about cross-learning, which is denoted $\chi^i_{jk,t}$ in Section 2 and summarizes the household's subjective model.[14] In the analysis below, I will refer to a respondent answering that inflation would make the economy stronger/little difference/weaker as having a positive/neutral/negative subjective model of inflation respectively.

There are two possible interpretations of this question. Households may view it as asking about the causal effects of inflation on the economy (as in the model of Spiegler, 2021). Alternatively, they could see it as asking about the most likely source of a rise in inflation, if they believe supply- and demand-driven inflation is associated with different real outcomes (Kamdar, 2019). For the purposes of this section, this distinction does not matter. In the decomposition of aggregate actions (Proposition 2), $\chi^i_{jk,t}$ is simply the degree to which households update their expectations of one variable when their expectation of another changes. In this case, it is the updating of expectations about the strength of the economy

[14] In Section 2.2 I noted that $\chi^i_{jk,t}$ comprised subjective models and any weighting the agent put on expectations of z^i_{kt}. Since these weights do not change the sign of $\chi^i_{jk,t}$, the qualitative responses to Question 1 still reflect the sign of the cross-learning from expected inflation to expectations of the state of the real economy, as long as no household perfectly observes the 'overall state of the economy'.

when expected inflation rises. The sign of this updating is captured by the question, whether it occurs because of a perceived direct causal link from inflation to the real economy, or a belief about the type of shocks hitting the economy.[15]

The next set of novel questions concern the information households use to form their inflation expectations, without asking what those expectations are. This allows us to learn about household information ($\delta^i_{k,t}$) without contamination from cross-learning ($\chi^i_{jk,t}$).

Question 2a *What were the most important factors in getting to your expectation for how prices in the shops would change over the next 12 months?*

Please select up to 4:

1. *How prices have changed in the shops recently, over the last 12 months*
2. *How prices have changed in the shops, on average, over the longer term i.e the last few years*
3. *Reports of current inflation in the media*
4. *Discussion of the prospects for inflation in the media*
5. *The level of interest rates*
6. *The inflation target set by the government*
7. *The current strength of the UK economy*
8. *Expectations about how economic conditions in the UK are likely to evolve*
9. *Other factors*
10. *None*

We can divide the possible answers into four categories. First, options 1 and 2 concern past experienced price rises. Options 3 and 4 are direct information about inflation. Options 5-8 concern other macroeconomic variables, either current or expected, and options 9 and 10 are extras. A rational household may well use the information sources in options 1,2 and 5-9 to forecast inflation, but in the decomposition in Proposition 2 this would represent cross-learning from information about other variables. To use the level of interest rates (5) to forecast inflation, for example, a household must employ a model of how interest rates relate to inflation. Similarly, to use past experienced price changes (1-2), households need a model of the persistence of inflation.[16] The only answers that represent the use of direct

[15] The distinction will matter when using a structural model to analyse counterfactual implications of this data. I therefore return to this issue in Section 4.

[16] Macaulay and Moberly (2022) find this perceived persistence is very heterogeneous across households. Note that strictly, option 3 also concerns past price changes, so the assumption here is that media reports of inflation tend to discuss both current and future inflation simultaneously. Appendix C.2 shows that the results below are robust to various small changes to this definition of the information indicator.

information about inflation are options 3 and 4.

Question 2a was only asked in 2016Q1, but very similar questions were asked at other times. In each, the respondent is asked about the information sources they used to arrive at their expected inflation, or that led them to change that expectation over the previous year. For each such question I construct a dummy variable equal to 1 if the respondent reports using direct information about inflation, and equal to 0 if they do not. Full details of these questions, and the options representing direct information, are in Appendix B. Combining these dummy variables gives an indicator for if the respondent used direct information on inflation in forming their expectations, that is whether $\delta^i_{\pi,t} > 0$. This indicator is observed for 8 separate quarters between 2009Q1-2019Q1. I confirm below that the key results of this section do not vary substantially with the changes in question wording over these periods.

In Appendix C.1 I confirm that these measures of information and subjective models correlate with questions on planned household consumption, and that the signs of these correlations are consistent with the measures picking up the desired elements of household beliefs. A further possible test of the information indicator would ask if households who obtain direct information about inflation make more accurate forecasts. However, if beliefs about the level of inflation affect subjective models, that may in turn change the incentives to acquire further information, complicating the predicted correlation between information and forecast accuracy. For this reason I leave discussion of this test for Section 6, after the model has been developed. The results are consistent with the model, adding further evidence that the information indicator reliably measures the object of interest.

The other questions used in this section are standard, asking households to give point estimates for "how prices have changed over the last twelve months" and "how much would you expect prices in the shops generally to change over the next twelve months". For each question, respondents choose from a list of ranges, and follow-up questions may then asked with more precise ranges, until the respondent has selected a 1 percentage-point bin between -5% and +10%, or end ranges $\leq -5\%, \geq 10\%$.

For the exercises in Section 3.4, I code perceptions and expectations at the midpoint of the selected bin, with the lowest and highest bins coded as -5.5% and 10.5% respectively. I refer to these answers as perceived and expected inflation respectively.

3.2 Information and subjective models in the cross-section

The first empirical result concerns the cross-sectional distribution of information and subjective models, the key relationship in the narrative heterogeneity channel. Table 1 shows the estimated average marginal effects from a probit regression of the information indicator defined in Section 3.1 on the respondent's subjective model of inflation, represented by their answer to Question 1. The first column shows this with quarter fixed effects only, while the second also includes a range of household controls.[17]

Table 1: Information correlates with subjective models

	(1)	(2)
end up stronger	-0.0102	-0.00827
	(0.0191)	(0.0192)
make little difference	-0.0356***	-0.0315**
	(0.0128)	(0.0129)
dont know	-0.0627***	-0.0605***
	(0.0172)	(0.0172)
Controls	None	All
Time FE	Yes	Yes
Observations	8270	8270

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Note: The table reports the average marginal effects from estimating a probit regression of the information indicator on the responses to Question 1. The information indicator equals 1 if the household reports using a direct source of information about inflation when forming their expectations, as defined in Appendix B. The omitted category is the belief that inflation makes the economy weaker. All regressions are weighted using the survey weights provided in the IAS.

Those answering that inflation makes no difference to the aggregate economy, or who don't know the effect of inflation, are significantly less likely to use information about inflation than someone who believes inflation makes the economy weaker. There is no significant difference in the probability of using direct inflation information between those holding this view and those with positive subjective models of inflation. The probability of using direct inflation information is 3-3.5 percentage points lower for those with a neutral model of the effects of inflation than those who believe inflation weakens the economy. Over the whole population 23% of respondents use direct inflation information, so this difference is non-trivial. More important than the magnitude, however, is that this shows a systematic cross-sectional relationship between information and subjective models, indicating a role for

[17]These are gender, age, class, employment status, income, education, region, and home-ownership status. Age, class, income and education are all reported in bands, and included as categorical variables.

the narrative heterogeneity channel. Assessing the quantitative relevance of this requires a model, as developed in Sections 4-6 below.

Empirical Result 1 *Households who believe inflation makes no difference to the economy acquire less information about inflation on average than households who believe inflation does affect the economy (in either direction).*

The information indicator is composed of answers to several slightly different questions. In particular, some questions concern information used to arrive at the respondent's point expectation for inflation, while others concern the information they used in changing those expectations over the last year. Most questions concern expected inflation over the next 12 months, but a minority ask about a longer horizon. In Appendix C.2 I repeat the regressions of Table 1 on subsets of the questions, and find the results are robust to these alternatives. As some respondents do not answer the unique survey questions used here, I also account for the concern that there may be selection bias in whose answers are observed, using a selection model as in Heckman (1979). Again, the results are robust.

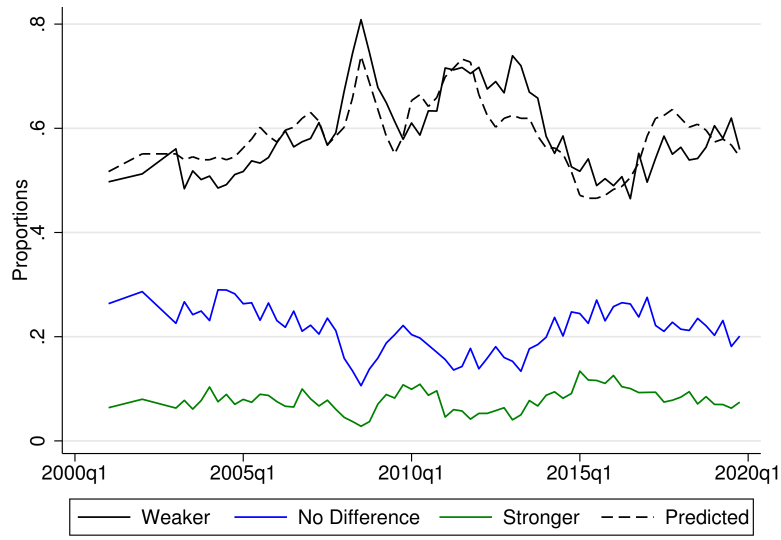
Result 1 is not consistent with models with exogenous information, as there would be no reason for information to be systematically correlated with household subjective models. It is, however, consistent with models of endogenous information acquisition, as the value of inflation information is lower for households who believe inflation makes little difference to other variables that matter for their decisions. The (broadly defined) strength of the aggregate economy is such a variable as long as households believe there is some relationship between the aggregate economy and their personal decisions, which is supported by evidence in Roth and Wohlfart (2020), among others. The implications of this link from subjective models to information acquisition are discussed further in Section 4.

3.3 Subjective models over time

I next turn to the time-series behavior of subjective models of the effects of inflation. Figure 1 shows the proportions answering Question 1 with each subjective model of inflation over time ('don't know' omitted for figure clarity).

The majority of households answer that inflation would make the economy weaker in all quarters, in keeping with the findings in Shiller (1997), Kamdar (2019), and Andre et al. (2022b). Combined with Empirical Result 1, this suggests that the covariance of information on inflation and cross-learning from inflation to the strength of the economy is negative. If

Figure 1: Proportions giving each answer to Question 1: "If prices started to rise faster than they are now, do you think Britain's economy would end up stronger, or weaker, or would it make little difference?"



Note: Proportions shown are calculated using the survey weights provided in the IAS. Proportion answering 'Don't know' is omitted for figure clarity. The dashed line is the predicted values from regressing the proportion reporting that inflation makes the economy weaker on annual CPI inflation: $\widehat{\Pr(\text{weaker})}_t = 0.057 \times \text{CPI inflation}_t + 0.466$. The coefficient on inflation is significant at the 1% level.

households consume more when they believe the economy is strong, the narrative heterogeneity channel will therefore reduce the consumption response to inflationary shocks.

The relatively long time series of the IAS also allows us to see that the distribution of answers varies substantially over time, and that much of this variation can be explained by recent inflation experiences. The correlation between annual CPI inflation and the proportion of respondents with negative models of inflation is extremely high, at 0.799. The dashed line in Figure 1 plots the predicted values from regressing this proportion on CPI inflation, showing that this correlation is strong across the whole sample. Tests in Appendix C.3 show that the correlation is robust to the addition of various macroeconomic controls, which themselves explain far less of the variation in the distribution of responses than realized inflation. The correlations are also robust to using inflation measures split by various household characteristics, to get closer to the rate of inflation in each household's own basket of goods. Finally, the proportions giving all other answers are also shown to be significantly negatively correlated with current inflation.

Empirical Result 2 *A greater proportion of households believe inflation weakens the economy when realized inflation is high.*

This is not what we would observe if households hold rational expectations. The question is about the effect of an aggregate variable (inflation) on the aggregate performance of the economy. Even if households are differentially exposed to the shock, if they all had model-consistent beliefs they would all give the same answer to this question. The fact that there is heterogeneity at all is evidence that at least some household subjective models are inconsistent with rational expectations.

The time-series patterns also suggest that the majority of households are not using New Keynesian-style models. In a textbook New Keynesian model, a rise in inflation causes the central bank to raise the nominal interest rate. If the Taylor Principle is satisfied, the real interest rate rises, so output falls. If it is not, the real rate falls, and output rises. If most households used this model, they should respond that inflation would make the economy weaker in the periods before interest rates reached the Zero Lower Bound, and they should switch to the view that inflation would make the economy stronger once we reach the ZLB in 2009. There is little evidence for this in Figure 1, and indeed statistical tests in Appendix C.3 find no evidence of such a shift.[18]

3.4 Inflation perceptions, expectations, and subjective models

Finally, I compare perceived and expected inflation across households with different subjective model beliefs. Figure 2 shows the time series of mean perceived and expected inflation by qualitative subjective model of inflation.

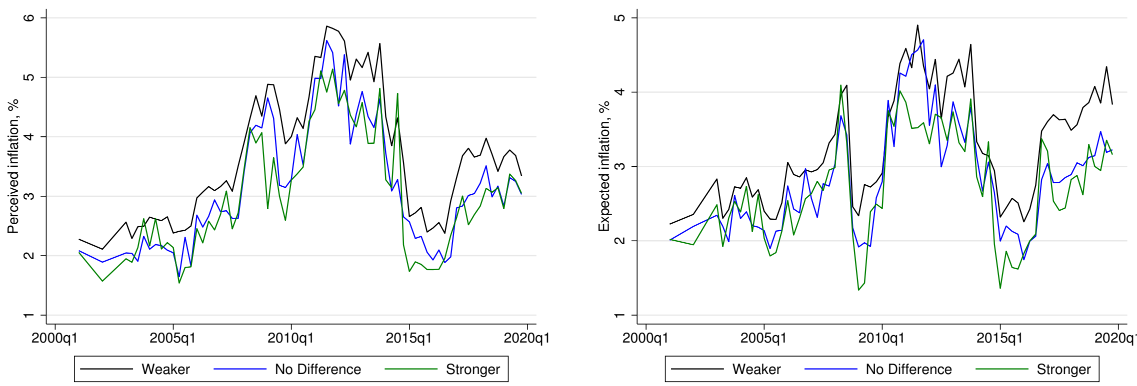
There are persistent differences between the perceptions and expectations of the different groups. Respondents who believe inflation weakens the economy systematically perceive that inflation has been higher, and expect it to be higher over the next year, than those who believe inflation makes no difference to the economy. They, in turn, perceive and expect higher inflation than those with positive subjective models of inflation.[19]

The differences are large: Table 2 shows that even after controlling for the full set of available household characteristics, those with a negative model of inflation perceive that

[18] This argument supposes that at least some households believe cost-push shocks are part of the drivers of inflation. While demand-driven inflation in a New-Keynesian model is associated with higher output whatever the monetary regime, we would still see some shifts in answer distributions at the ZLB if cost-push shocks are perceived to occur with positive probability. In principle, after 2009 a New Keynesian model would predict that a sufficiently large rise in inflation would lift the economy away from the ZLB, implying higher real interest rates and lower output. However, in 2013 the Bank of England began forward guidance committing to maintaining low interest rates, so it is unlikely that households were expecting them to contract in response to small rises in inflation at this time.

[19] Dräger et al. (2020) similarly find for German households that inflation expectations are higher among those reporting that they would prefer inflation to be lower.

Figure 2: Inflation perceptions and expectations by subjective model.



(a) Perception, past 12 months: $\mathbb{E}_t \pi_{t,t-12}$ (b) Expectation, next 12 months: $\mathbb{E}_t \pi_{t+12,t}$

Note: Perceived inflation refers to beliefs about what inflation has been over the past 12 months, and expected inflation refers to expectations for the next 12 months. Averages for each variable are calculated using the survey weights provided in the IAS. Average perceptions and expectations among respondents answering 'Don't know' to the subjective model question (Question 1) are omitted for figure clarity.

inflation has been 54 basis points higher than those with a neutral model, and 70 basis points higher than those with a positive model. The gaps are similarly large and strongly significant for expected inflation. Appendix C.4 shows that these results are not driven by selection bias from missing observations for inflation perceptions and expectations.

Empirical Result 3 *Households who believe inflation weakens the economy on average perceive higher current inflation, and expect higher future inflation, than those with less negative subjective models.*

This is not driven by the households using different kinds of information to arrive at their perceptions and expectations: Table 1 shows that the households with positive subjective models use similar information sources to those with negative models. It is, however, consistent with information about high inflation causing households to update their subjective models towards more negative views. Although the exercises here do not identify the direction of causation, such a mechanism can simultaneously account for Results 2 and 3. Within a period, those who receive signals that inflation is high shift to more negative subjective models, and when realized inflation rises more households receive such signals. This is explored in detail in Section 4.

Table 2: Perceived and expected inflation are higher for those with more negative subjective models.

	(1) Perceived inflation	(2) Expected inflation
end up stronger	-0.696***	-0.565***
	(0.0371)	(0.0353)
make little difference	-0.543***	-0.466***
	(0.0226)	(0.0207)
dont know	-0.462***	-0.413***
	(0.0315)	(0.0294)
Controls	Yes	Yes
Time FE	Yes	Yes
R-squared	0.179	0.113
Observations	85803	85201

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Note: The table reports the results of regressing perceived and expected inflation on respondent subjective models (responses to Question 1) . The omitted category is the belief that inflation makes the economy weaker. All regressions are weighted using the survey weights provided in the IAS.

4 Model

In this section I build a model with heterogeneous information and subjective models around inflation. This will be used to explore the implications of the empirical results above for aggregate shock transmission through the narrative heterogeneity channel.

4.1 Households

Time is discrete, and the period is denoted by t. The economy is populated by a measure 1 of households. Each period, household i chooses consumption C_t^i to maximize expected discounted utility:

$$\tilde{\mathbb{E}}_0^i U_0^i = \tilde{\mathbb{E}}_0^i \sum_{t=0}^{\infty} \beta^t \frac{(C_t^i)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \tag{17}$$

subject to:

$$C_t^i + B_t^i = \frac{R_{t-1}}{\Pi_t} B_{t-1}^i + Y_t \tag{18}$$

where Y_t is the real income received by all households in period t, R_t is the gross nominal interest rate on one-period bonds B_t^i, and Π_t is gross inflation between periods $t-1$ and t. $\beta \in (0,1)$ is the discount factor, and $\sigma > 0$ is the elasticity of intertemporal substitution. Income and prices are observed before the consumption choice in period t, but future income and prices are unknown. The operator $\tilde{\mathbb{E}}_t^i$ reflects the expectations of household i in period t, which may not coincide with rational expectations. However, given their subjective model for the evolution of R, Π, Y, the household uses their information optimally. Any non-rationality in expectations therefore comes only from misperceptions in these laws of motion.

While households observe the current price level, I assume that they may not perfectly observe the current rate of inflation. This assumption is common in models with noisy information (e.g. Coibion and Gorodnichenko, 2015), and is consistent with the evidence in Macaulay and Moberly (2022), who find substantial uncertainty about current inflation.[20]

The first order condition is a standard consumption Euler equation:

$$(C_t^i)^{-\frac{1}{\sigma}} = \beta \tilde{\mathbb{E}}_t^i \frac{R_t}{\Pi_{t+1}} (C_{t+1}^i)^{-\frac{1}{\sigma}} \tag{19}$$

To proceed, I take a log-quadratic approximation to utility, as is common in the rational inattention literature (e.g. Maćkowiak and Wiederholt, 2009). The approximation is taken about a steady state with $\Pi = 1, R = \beta^{-1}$.

Lemma 1 *Let $\tilde{\mathbb{E}}_0^{i*} U_0^{i*}$ denote the expected utility of an otherwise identical household who observes Π_t precisely before choosing C_t^i. Furthermore, let $\hat{U}_0^{i*}$ and $\hat{U}_0^i$ denote the log-quadratic approximation to the discounted utility of the fully-informed and uninformed households respectively. The expected utility loss from imperfect information about Π_t is:*

$$\tilde{\mathbb{E}}_0^i(\hat{U}_0^{i*} - \hat{U}_0^i) = \frac{(\bar{C}^i)^{1-\frac{1}{\sigma}}}{2\sigma} \tilde{\mathbb{E}}_0^i \sum_{t=0}^{\infty} \beta^t (c_t^i - c_t^{i*})^2 \tag{20}$$

where lower-case letters are log-deviations of the corresponding variables from steady state, and c_t^{i*} denotes the period-t consumption of the fully-informed household.

Proof. Appendix D.1 ■

Note that the fully-informed household invoked in Lemma 1 uses the same potentially non-rational expectations operator as the uninformed household. That is, they have the

[20] One way to microfound this is to assume that households consist of a forecaster, who forms expectations without observing current inflation, and a shopper who uses those forecasts (along with observed current prices) to make consumption decisions. A similar assumption is made in Pfäuti (2022).

same subjective model, but different information.

To focus on the feedback between subjective models and information choices, I take steady state assets $\bar{B}^i \to 0$.[21] With this assumption, the problem of a fully-informed household is identical to that in Appendix A.2, and so their consumption function is:

$$c_t^{i*} = (1-\beta)\sum_{s=0}^{\infty}\beta^s \tilde{\mathbb{E}}_t^{i*} y_{t+s}^i - \sigma\beta\sum_{s=0}^{\infty}\beta^s(\tilde{\mathbb{E}}_t^{i*} r_{t+s} - \tilde{\mathbb{E}}_t^{i*}\pi_{t+s+1}) \tag{21}$$

Since utility losses from deviating from this are quadratic, a household with imperfect information sets $c_t^i = \tilde{\mathbb{E}}_t^i c_t^{i*}$.

4.2 Firms

As the focus of this model is the behavior of households, I keep the production side simple. This allows for analytic solutions in the analysis below.

Monopolistically competitive intermediate goods producers set prices subject to quadratic adjustment costs, and produce using labor as the only input. They supply a perfectly competitive final goods producer, who combines the intermediate goods varieties with a CES production function. Log-linearizing the solution to the intermediate goods firm pricing problem about the zero-inflation steady state yields the Phillips Curve:

$$\pi_t = \beta\mathbb{E}_t\pi_{t+1} + \kappa mc_t \tag{22}$$

where mc_t is the real marginal cost of intermediate goods producers in period t, and κ is a combination of model parameters. As this is standard in many models and textbooks, I leave the derivation and details to Appendix D.2. I also maintain the textbook assumption of rational expectations for firms.

Labor supply is determined by a continuum of unions, who each supply differentiated labor varieties to a perfectly competitive labor packer (as in e.g. Schmitt-Grohé and Uribe, 2005; Auclert et al., 2018). Households supply as much labor as is demanded by the unions, who set nominal wages for their labor variety. Labor supply therefore has the same structure as the supply of final goods.

In New Keynesian models with labor markets of this kind, unions are typically subject

[21] Michelacci and Paciello (2020) show that with ambiguity aversion, wealth heterogeneity implies heterogeneity in subjective models. Combining this with endogenous information choices, wealth could therefore form an additional reason for a systematic relationship between information and subjective models. This is beyond the scope of this paper.

to frictions in the setting of nominal wages. I depart from this, and instead assume that it is real wages which are sticky, as in e.g. Blanchard and Galí (2007). Furthermore, I take this stickiness to the limit where real wages are perfectly rigid. This simplification ensures that inflation is entirely supply-driven, as the only fluctuations in real marginal costs come from changes in TFP. This substantially aids tractability, particularly as the key exercises below concern the consequences of shocks to inflation, and these assumptions eliminate any feedback from those consequences back into inflation. The lack of demand effects on inflation is discussed further in Section 4.5.

All firm profits are transferred back to households as a lump sum, so all revenues are returned to households either through wages or dividends. Costs due to price adjustment costs drop out when the model is log-linearized. To first order, real income is therefore equivalent to real output.

4.3 Policy and market clearing

Nominal interest rates are set according to a simple Taylor rule:

$$r_t = \phi \pi_t + v_{rt} \tag{23}$$

where ϕ is a constant, and $v_{rt} \sim N(0, \sigma_r^2)$ is a monetary policy shock.

Goods market clearing implies that all output is consumed by households each period:

$$y_t = \bar{c}_t \tag{24}$$

where $\bar{c}_t = \mathbb{E}_I(c_t^i)$ is aggregate consumption across households.

Finally, an exogenous AR(1) process for TFP, combined with equation 22 and fixed real wages, implies:

$$\pi_t = \rho_\pi \pi_{t-1} + v_{\pi t} \tag{25}$$

where the persistence parameter $\rho_\pi \in [0, 1)$, and $v_{\pi t} \sim N(0, \sigma_a^2)$ is driven by an exogenous TFP shock. The full derivation of this is provided in Appendix D.2.

4.4 Temporary equilibrium

This model fits into the framework of Section 2. Consumption c_t^i is the choice variable x_t^i, so the consumption function given by taking expectations over equation 21 maps into equation

1. The vector of relevant variables $\boldsymbol{z}_t$ therefore consists of $\{y_{t+s}, r_{t+s}, \pi_{t+s+1}\}_{s=0}^{\infty}$. Equations 23, 24, 25 form the general equilibrium conditions, mapping into equation 14.

Equilibrium definition. Given exogenous shocks $v_{rt}, v_{\pi t}$ and household expectations $\{\tilde{\mathbb{E}}_t^i y_{t+s}, \tilde{\mathbb{E}}_t^i r_{t+s}, \tilde{\mathbb{E}}_t^i \pi_{t+s}\}_{s=0}^{\infty}$, a temporary equilibrium consists of $\{c_t^i, r_t, \pi_t, y_t\}$ such that:

1. *Households:* households choose consumption c_t^i to maximize expected lifetime utility (minimizing equation 20).
2. *Firms:* firms set prices to maximize expected lifetime profits, implying inflation π_t follows the process in (25).
3. *Monetary Policy:* policymakers choose the nominal interest rate r_t according to (23).
4. *Market Clearing:* the goods market clears (y_t satisfies equation 24).

4.5 Expectations

Households form expectations by taking information on each variable and forecasting forward using their subjective models. Their period-t information set consists of the history to period t of observations of r_t, y_t, and any signals observed about π_t. The information set of the hypothetical fully-informed agent also includes the history to period t of realizations of π_t.

Subjective models take the following form:

$$\pi_t = \rho_\pi^i \pi_{t-1} + u_{\pi t} \tag{26}$$

$$r_t = \phi^i \pi_t + u_{rt} \tag{27}$$

$$y_t = \alpha^i \pi_t + \lambda^i r_t + \rho_y^i y_{t-1} + u_{yt} \tag{28}$$

where $u_{xt} \sim N(0, \sigma_x^2)$ for $x \in \{\pi, r, y\}$, and $\rho_\pi^i, \rho_y^i \in (0, 1)$.

The subjective models for π_t, r_t therefore have the same functional forms as the data generating processes for those variables (equations 25 and 23). The equilibrium data generating process for y_t is derived below: the functional form of equation 28 is a tractable approximation to that equilibrium law of motion. Note, however, that the parameters of these subjective models may differ across households. Since π_t, r_t, y_t are aggregate variables, this heterogeneity rules out the possibility that all households have rational expectations.

To add to their observations of r_t, y_t, each household receives an idiosyncratic signal about current inflation, of the form:

$$s_t^i = \pi_t + \varepsilon_t^i, \quad \varepsilon_t^i \sim N(0, \sigma_{\varepsilon i}^2) \tag{29}$$

For simplicity, I assume that households do not infer anything about π_t from observed y_t and r_t. In principle, these are also noisy signals about π_t, but households do not make use of them when forming perceptions of current inflation. This is consistent with the rational inattention microfoundation offered for the household information structure in Section 4.6.[22] Households therefore form inflation perceptions using a standard Kalman filter:

$$\tilde{\mathbb{E}}_t^i \pi_t = K^i(\pi_t + \varepsilon_t^i) + (1 - K^i)\rho_\pi^i \tilde{\mathbb{E}}_{t-1}^i \pi_{t-1} \tag{30}$$

As the initial subjective models are constant for each household, so too is $\sigma_{\varepsilon i}^2$. I therefore assume that each household uses the steady-state Kalman gain:

$$K^i = \frac{\tilde{\text{Var}}^i(\pi_t | \mathcal{I}_{t-1}^i)}{\sigma_{\varepsilon i}^2 + \tilde{\text{Var}}^i(\pi_t | \mathcal{I}_{t-1}^i)} \tag{31}$$

where $\mathcal{I}_{t-1}^i$ is the information set of household i in period $t-1$.

Unlike in Section 3, this specification of expectations does restrict the interpretation of Question 1 in the IAS. The only shock perceived to be driving inflation is $u_{\pi t}$, so there is no room for disagreement about the source of inflation shocks. Heterogeneous cross-learning from inflation to the real economy can only therefore come from heterogeneous beliefs about the causal effects of inflation. In the model, this is consistent with the true law of motion for inflation (equation 25).

This assumption aids tractability, but also reflects the fact that the distribution of survey answers is very consistent over time, in levels and in how it correlates with realized inflation. If the answers reflected beliefs about the type of shocks driving inflation, this distribution would change across time periods characterized by different types of shocks. Since the distribution of subjective models evolved in the same way with the run-up in inflation before the Great Recession and the currency devaluation-driven spike after the Brexit referendum, it does not appear that the source of inflation shocks plays a key role in the majority of survey answers. Finally, this formulation is also consistent with existing literature finding household

[22] Strictly, rational inattention models assume agents choose among all possible signals. So y_t and r_t are available signals, but the household chooses not to pay to process them when forming their inflation perception.

inflation expectations are well-described by such simple forecasting rules (e.g. Adam, 2007).

With this setup, the expectations of a fully-informed household are (derivation in Appendix D.3):

$$\tilde{\mathbb{E}}_t^{i*}\pi_{t+s} = (\rho_\pi^i)^s \pi_t \tag{32}$$

$$\tilde{\mathbb{E}}_t^{i*}r_{t+s} = \phi^i(\rho_\pi^i)^s \pi_t \tag{33}$$

$$\tilde{\mathbb{E}}_t^{i*}y_{t+s} = \frac{(\alpha^i + \lambda^i\phi^i)\rho_\pi^i}{\rho_\pi^i - \rho_y^i}\big((\rho_\pi^i)^s - (\rho_y^i)^s\big)\pi_t + (\rho_y^i)^s y_t \tag{34}$$

Substituting these into the consumption function (21), and taking expectations, the consumption function of household i is:

$$c_t^i = \frac{1-\beta}{1-\beta\rho_y^i}y_t - \sigma\beta r_t + \frac{\beta\rho_\pi^i[(1-\beta)(\alpha^i+\lambda^i\phi^i) - \sigma(\phi^i\beta - 1)(1-\beta\rho_y^i)]}{(1-\beta\rho_\pi^i)(1-\beta\rho_y^i)}\tilde{\mathbb{E}}_t^i\pi_t \tag{35}$$

Given this, and market clearing (24), the equilibrium process for y_t is:

Proposition 4 *The data generating process for y_t in equilibrium, under household consumption functions (35), is given by:*

$$y_t = \alpha\pi_t + \lambda r_t + \rho_y \int \hat{\omega}^i \tilde{\mathbb{E}}_{t-1}^i \pi_{t-1} di \tag{36}$$

where $\alpha, \lambda, \rho_y, \hat{\omega}^i$ are combinations of model parameters, defined in Appendix D.4.

Proof. Appendix D.4. ■

This has the same functional form as the household subjective model for y_t (equation 28), except that the household believes the persistence in y_t comes from y_{t-1}, rather than a lagged weighted average of inflation perceptions. However, in Appendix D.4 I show that these terms are closely related, as both are weighted averages of past inflation and household prior beliefs about inflation. The subjective model (28) is not exactly consistent with the true process, because the weightings on these components differ somewhat across the two cases. Equation 28 is therefore an approximation to the true functional form of the y_t law of motion, which will be substantially more tractable going forwards, as it eliminates the need to consider higher-order beliefs. The assumption in equation 28 is therefore that households do not take into account that real incomes depend on the expectations of others, which they do not observe, but rather assume the persistence comes from lagged income, which they do.

4.6 Fitting the survey results

I now add assumptions on the distributions of information and subjective models across households to match the correlations observed in the survey data in Section 3.

Specifically, to match Result 1, I calibrate the variance of each household's signal s_t^i as a function of their subjective model. To match Results 2 and 3, I then assume that the perceived relationship between inflation and real income (α^i) can vary with the household's perception of inflation ($\tilde{\mathbb{E}}_t^i \pi_t$). The timing is therefore as follows: a household starts the period with an initial subjective model, which determines the precision of their inflation signal. After observing the realization of that signal, the household forms a perception of inflation, which they use to update their subjective model. The realized signals and updated subjective model are then combined to form the expectations used to choose consumption. In this decision I use the anticipated utility assumption, common in models with learning, that households act in period t as if they are certain that their subjective model will not change in future periods (see e.g. Bullard and Suda, 2016). For now, the initial subjective models at the start of each period are fixed over time for each household. This aids the exposition of the core mechanisms, and is relaxed in Section 6.

The qualitative pattern in Result 1 is that households who believe inflation makes little difference to the economy use less information about inflation. To replicate this in the model, I assume $\sigma^2_{\varepsilon i}$ is such that:

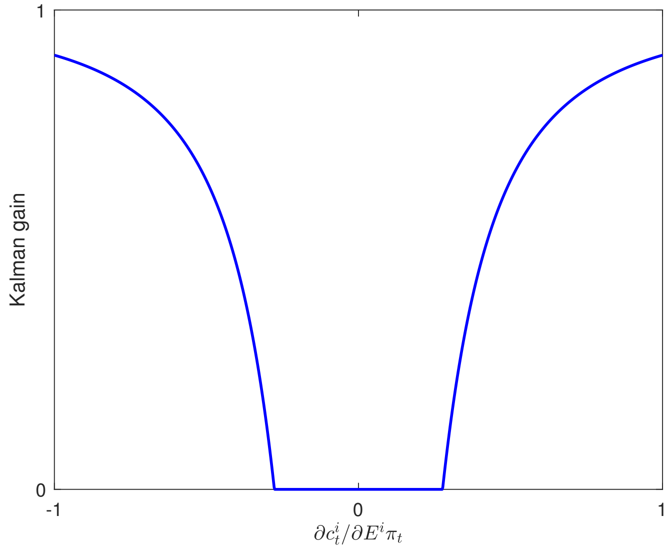
Information properties. For some positive threshold δ^*:

1. $K^i = 0$ if $\left(\dfrac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t}\right)^2 < \delta^*$.
2. K^i is strictly increasing in $\left(\dfrac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t}\right)^2$ if $\left(\dfrac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t}\right)^2 \geq \delta^*$.
3. $K^i \to 1$ as $\left(\dfrac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t}\right)^2 \to \infty$.

That is, households with a subjective model which implies inflation has little effect on their consumption decision receive no information about inflation. Households with subjective models in which inflation has larger effects on decisions, positively or negatively, observe more precise signals, implying larger Kalman gains. In the limit, households with subjective models in which inflation has extreme effects observe perfectly precise information, and become fully informed. These properties can be seen graphically in Figure 3.

A simple proxy for 'the strength of the economy' might be aggregate consumption. If

Figure 3: K^i against the elasticity of consumption to perceived inflation. Calibration: Appendix F.



households believe others hold beliefs similar to their own, then the households who report in the survey that inflation makes no difference to the economy are those with $\partial c_t^i/\partial\tilde{\mathbb{E}}_t^i\pi_t$ is close to zero.[23] These assumptions therefore fit the model to empirical Result 1.

Results 2 and 3 imply that more negative subjective models are associated with higher realized and perceived inflation, in the time-series and cross-section respectively. To replicate this in the model, I allow the household's perception of how inflation affects real income in the subjective model (α^i) to vary:

Subjective model properties. Denote the parameter value used in determining information precision at the start of the period as α_0^i, and the updated parameter used to make consumption decisions in period t as $\hat{\alpha}_t^i$. This parameter is such that:

1. $\hat{\alpha}_t^i = \alpha_0^i$ if $\tilde{\mathbb{E}}_t^i\pi_t = 0$.
2. $\dfrac{\partial\hat{\alpha}_t^i}{\partial\tilde{\mathbb{E}}_t^i\pi_t} < 0$ for all households i.

That is, if a household believes inflation is at steady state, they do not update their subjective model. However, perceiving inflation above steady state will cause them to distort update their subjective model towards the view that inflation erodes real income. Perceptions below steady state have the opposite effect.

[23]See Dräger et al. (2020) for evidence that household beliefs about what is good for the economy overall and for them personally are highly correlated.

These properties immediately imply that higher perceived inflation is associated with a lower $\partial c_t^i/\partial\tilde{\mathbb{E}}_t^i\pi_t$, as $\partial c_t^i/\partial\tilde{\mathbb{E}}_t^i\pi_t$ is increasing in $\hat{\alpha}_t^i$ (equation 35). Similarly, higher realized π_t implies weakly higher $\tilde{\mathbb{E}}_t^i\pi_t$ for all households, implying more households have low values of $\partial c_t^i/\partial\tilde{\mathbb{E}}_t^i\pi_t$. Under the interpretation that answers to Question 1 in the survey reflect these reactions to perceived inflation, the properties above therefore match Results 2 and 3. Formal statements of these results are provided in Appendix D.5.

Microfoundations and functional forms. For calibrating the model to the survey data, these reduced-form assumptions are adequate. However, for the intuitions behind the results in Sections 5 and 6, and to put functional forms to the reduced-form assumptions, it is useful to provide microfoundations of the reduced-form patterns.

The information properties arise when it is costly for households to process information about current inflation, as in the literature on rational inattention (Maćkowiak et al., 2020). In Appendix D.5 I show that a household facing such costs optimally chooses signals of the form in Equation 29, with noise variances such that:

$$\begin{cases} K^i = 0 & \text{if } \left(\dfrac{\partial c_t^i}{\partial\tilde{\mathbb{E}}_t^i\pi_t}\right)^2 < \delta^{i*} \\ \dfrac{1-K^i}{(1-(\rho_\pi^i)^2(1-K^i))^2} = \dfrac{\delta^{i*}}{(1-(\rho_\pi^i)^2)^2}\left(\dfrac{\partial c_t^i}{\partial\tilde{\mathbb{E}}_t^i\pi_t}\right)^{-2} & \text{if } \left(\dfrac{\partial c_t^i}{\partial\tilde{\mathbb{E}}_t^i\pi_t}\right)^2 \geq \delta^{i*} \end{cases} \tag{37}$$

where δ^{i*} is a threshold defined in Appendix D.5.

This is consistent with the reduced-form properties stated above. Intuitively, households who intend to respond strongly to information about inflation place the most value on that information, and so process more of it. In all exercises below I assume K^i follows this form.

For the subjective model properties, there are several possible microfoundations. For example, if households believe there is an optimal level of inflation, such that real income is increasing in inflation below that bliss point, but is decreasing beyond it, their subjective models would behave this way. Appendix D.5 provides an alternative, in which households are ambiguity averse, and face Knightian uncertainty about α^i. In that environment households distort their subjective model towards the worst case, which varies with perceived inflation.[24] This microfoundation implies that the updates to subjective models are linear

[24]This approach relates to that of Michelacci and Paciello (2020), who note that ambiguity aversion naturally generates the negative correlation between preferences and expectations I observe for inflation. Similarly, in Ilut et al. (2020) firm worst-case beliefs depend on the direction of price changes.

in inflation perceptions:

$$\hat{\alpha}_t^i = \alpha_0^i + \alpha_1^i \tilde{\mathbb{E}}_t^i \pi_t \tag{38}$$

where $\alpha_1^i < 0$. In all exercises below I assume $\hat{\alpha}_t^i$ follows this form.

5 Implications of narrative heterogeneity

I now analyse the effect of the narrative heterogeneity channel on the transmission of inflationary shocks. Calibrating the model to macroeconomic data from the UK, and the survey data, I find that the narrative heterogeneity channel is quantitatively important for the transmission of inflationary shocks.

5.1 Selection in attention

First, consider the effect of subjective models on information choice. To isolate this, assume for now that $\alpha_1^i = 0$, so the only heterogeneity in subjective models is that present at the start of each period, when households choose their information.

Consider a shock that increases inflation in period t. The effect on the consumption of household i on impact is:

$$\frac{dc_t^i}{d\pi_t} = \Theta^i \frac{dy_t}{d\pi_t} - \sigma\beta\phi + \frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t} K^i \tag{39}$$

where K^i gives the response of $\tilde{\mathbb{E}}_t^i \pi_t$ to π_t (equation 30), ϕ is the response of r_t (equation 23), and

$$\Theta^i = \frac{1-\beta}{1-\beta\rho_y^i} \in (0,1] \tag{40}$$

Aggregating across households and using the market clearing condition (24), the response of aggregate consumption is:

$$\frac{d\bar{c}_t}{d\pi_t} = \frac{1}{1-\bar{\Theta}} \left[\int_0^{P_0} \omega^i \frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t} K^i di - \sigma\beta\phi \right] \tag{41}$$

where ω^i is a weight on household i as in equation 11, $\bar{\Theta}$ is the (similarly-weighted) average Θ^i across households, and the households who pay no attention ($K^i = 0$) are indexed by $i \in [P_0, 1]$.

To see how the relationship between information and subjective models affects aggregate outcomes, compare this to a model in which all households have the same Kalman gain $\bar{K}$, equal to the average K^i from the baseline model:

$$\bar{K} = \mathbb{E}_I(K^i) = \mathbb{E}_I(K^i | K^i > 0) \cdot P_0 \tag{42}$$

This, for example, could reflect an economist calibrating a model with homogeneous information frictions to micro-level evidence on household information. In such a homogeneous-K model the aggregate response of consumption to the inflation shock can be decomposed as:

$$\left.\frac{d\bar{c}_t}{d\pi_t}\right|_{K^i=\bar{K}} = \frac{1}{1-\bar{\Theta}} \left[\int_0^{P_0} \omega^i \frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t} K^i \frac{\bar{K}}{K^i} di + \int_{P_0}^1 \omega^i \frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t} \bar{K} di - \sigma\beta\phi \right] \tag{43}$$

The first term is identical to that in the baseline model with endogenous attention (equation 41), except that each household's response is weighted by $\bar{K}/K^i$. Relative to the baseline model, the consumption responses of more attentive households receive a lower weight, while less attentive households are over-weighted.

The second integral concerns the consumption responses of inattentive households. In the baseline model, their response to perceived inflation is irrelevant, because their inflation perceptions are unaffected by the shock. Here, however, their perceptions react to the shock with elasticity $\bar{K}$. The least attentive households are also therefore over-weighted in the homogeneous-K model.

This leads to systematic differences in aggregate consumption responses, because the most attentive households in the baseline model have high K^i precisely because they respond strongly to perceived inflation. Formally, the difference between the aggregate consumption responses in the endogenous-K^i baseline and the homogeneous-K model is:

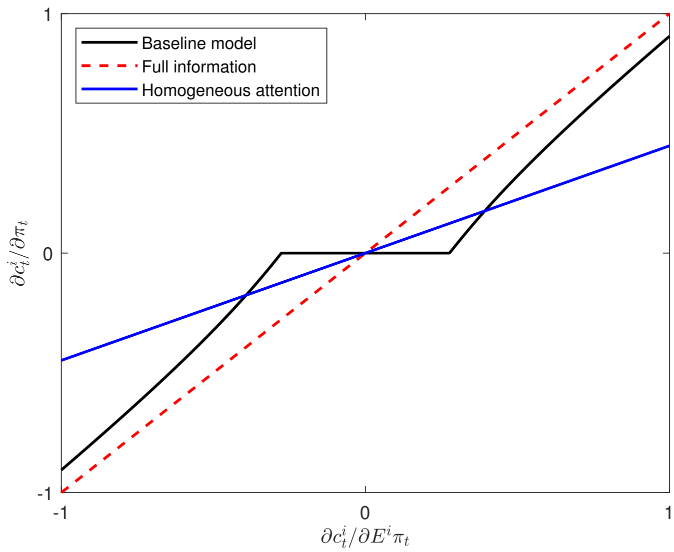
$$\frac{d\bar{c}_t}{d\pi_t} - \left.\frac{d\bar{c}_t}{d\pi_t}\right|_{K^i=\bar{K}} = \frac{1}{1-\bar{\Theta}} Cov_I\left(\frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t}, K^i \right) \tag{44}$$

The difference therefore depends on the covariance of information and subjective models: by making attention exogenous, the homogeneous-K model omits the narrative heterogeneity channel of shock transmission.[25] This covariance depends on the distribution of subjective models, as K^i is increasing in the absolute value of $\partial c_t^i / \partial \tilde{\mathbb{E}}_t^i \pi_t$. Among households with $\partial c_t^i / \partial \tilde{\mathbb{E}}_t^i \pi_t > 0$, the covariance of consumption responses and K^i is positive, but among those with $\partial c_t^i / \partial \tilde{\mathbb{E}}_t^i \pi_t < 0$ it is negative.

[25] Note there is no response heterogeneity channel because all households have the same policy functions. All heterogeneity in $\partial c_t^i / \partial \tilde{\mathbb{E}}_t^i \pi_t$ therefore comes from cross-learning from current inflation to expectations of other variables.

This implies that, for most distributions of subjective models, the narrative heterogeneity channel amplifies the aggregate consumption response to the shock, relative to the homogeneous-K model. Figure 4 shows this effect graphically. It plots the consumption response of an individual household to a shock to π_t, holding r_t, y_t fixed, against the same household's response to an increase in perceived inflation $\tilde{\mathbb{E}}_t^i \pi_t$. If households observed inflation precisely, this would simply be the 45° line (red dashed line).

Figure 4: Consumption response to a change in actual inflation against response to perceived inflation. Parameters listed in Appendix F.



The black solid line shows this relationship in the baseline model with endogenous K^i. Households with $\partial c_t^i / \partial \tilde{\mathbb{E}}_t^i \pi_t$ close to zero pay no attention to current inflation, and so their perceptions of inflation do not change when the shock hits. They therefore do not react. Households with greater $\partial c_t^i / \partial \tilde{\mathbb{E}}_t^i \pi_t$ pay more attention, so their perceptions are more sensitive to the shock, and their elasticity of consumption to π_t is closer to the 45° line.

If the endogenous K^i is replaced by a fixed $\bar{K}$ for all households, the elasticity of c_t^i to π_t is instead given by the blue solid line. Relative to the baseline model, consumption responses are drawn closer to the full-information line for all households with $\partial c_t^i / \partial \tilde{\mathbb{E}}_t^i \pi_t$ such that $K^i < \bar{K}$ in the baseline model. Conversely, consumption responses are reduced towards zero for all those who are more attentive than average in the baseline model. Since the less attentive households are the ones who would react the least under full information, removing the narrative heterogeneity channel in this way weakens the effect of the shock.[26]

[26] While this intuition dominates for most subjective model distributions, it is possible to construct cases in which the narrative heterogeneity effect instead attenuates aggregate transmission relative to the homogeneous-K^i case. These are discussed in Appendix D.6.

The equilibrium response of real income only amplifies this effect, as a smaller partial-equilibrium consumption response implies a smaller change in real income, further weakening consumption responses.

This is analogous to the selection effect in menu cost models of price setting (Caplin and Spulber, 1987; Golosov and Lucas, 2007). In those models, price adjustments are disproportionately drawn from firms desiring large price changes. Here, households obtaining information about inflation are disproportionately drawn from those who react strongly to that information.[27] Just as the price level in a menu cost model is more flexible than the average firm-level flexibility, this implies that aggregate consumption is typically more responsive to inflation than is implied by micro-level estimates of household attention. The narrative heterogeneity channel can therefore explain why representative-agent models typically require only small information frictions to match aggregate data (Maćkowiak and Wiederholt, 2015), while micro-level studies find very large degrees of inattention (Link et al., 2021).

A further implication concerns identification in information treatment experiments aimed at estimating the causal effects of expectations (see Candia et al., 2020, for a review). The standard approach in these studies is to regress an outcome variable on the expectation of interest, instrumented using an indicator for whether the respondent was treated.[28] The estimate is therefore consistent for the local average treatment effect on those who update their expectations as a result of the information provision, and is most influenced by those who update the furthest. The selection effect studied here suggests that those compliers will disproportionately be those with the smallest responses to information: they start out with the most uncertain beliefs due to their lack of attention, and so they update expectations the most when shown publicly available information. However, when a shock hits the economy, these are not the households whose expectations matter. It is the attentive households who observe the shock precisely, and react most strongly. Of course, in some settings the response of inattentive households is precisely the object of interest, e.g in central bank communication with the general public (Haldane et al., 2021; Coibion et al., 2022).

5.2 State-dependent shock transmission

I now return to the two-way feedback between information and subjective models. Restoring subjective model updating ($\alpha_1^i < 0$), the interaction between the two components of expec-

[27] Afrouzi and Yang (2021) study a similar mechanism, in which firms pay attention to aggregate variables only when they need to change prices.

[28] It is also common to use a second instrument, the interaction of the treatment indicator with the agent's prior expectation (e.g. Coibion et al., 2019). This does not substantially change the intuition discussed here.

tations implies that the transmission of inflation shocks to aggregate consumption depends on the size and recent history of realized inflation deviations from steady state. I begin by showing how the aggregate consumption response to an inflation shock depends on the distribution of inflation perceptions, before showing how that distribution varies with the size and history of inflation shocks.

The distribution of $\tilde{\mathbb{E}}_t^i \pi_t$. Substituting equation 38 into the consumption function (35) yields:

$$\left.\frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t}\right|_{\hat{\alpha}_t^i} = \left.\frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t}\right|_{\alpha_0^i} - \Omega^i \tilde{\mathbb{E}}_t^i \pi_t \tag{45}$$

where $\Omega^i > 0$ is a function of preference and subjective model parameters:

$$\Omega^i = -\frac{\beta(1-\beta)\rho_\pi^i \alpha_1^i}{(1-\beta\rho_\pi^i)(1-\beta\rho_y^i)} \tag{46}$$

Using this, we can decompose the aggregate consumption response to inflation (equation 41) as follows:

$$\begin{aligned}\frac{d\bar{c}_t}{d\pi_t} &= \frac{1}{1-\bar{\Theta}}\left[\int_0^1 \omega^i K^i \left.\frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t}\right|_{\alpha_0^i} di - \int_0^1 \omega^i K^i \Omega^i \tilde{\mathbb{E}}_t^i \pi_t di - \sigma\beta\phi\right] \\ &= \frac{1}{1-\bar{\Theta}}\left[\mathbb{E}_I\left(K^i \left.\frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t}\right|_{\alpha_0^i}\right) - \mathbb{E}_I(K^i)\mathbb{E}_I(\Omega^i \tilde{\mathbb{E}}_t^i \pi_t) - Cov_I(K^i, \Omega^i \tilde{\mathbb{E}}_t^i \pi_t) - \sigma\beta\phi\right]\end{aligned} \tag{47}$$

The first term of the aggregate elasticity to inflation is a function of underlying parameters only. Since the initial subjective models held by households at the start of each period are assumed to be fixed here, this is unaffected by realized shocks.

The second term, however, shows that the average subjective model will adjust towards lower values of $\hat{\alpha}_t^i$ as perceived inflation rises. This more negative average subjective model will reduce the aggregate consumption elasticity to inflation. The third term shows that such a rise in perceived inflation will have more of an effect if it occurs in households who process a lot of information about inflation. These are the time-varying components of the representative agent and narrative heterogeneity channels identified in Section 2.

Size dependence. Differentiating equation 47 with respect to current inflation, and using the Kalman filtering equation (30) to extract the response of perceived inflation, we obtain:

$$\frac{d}{d\pi_t}\left(\frac{d\bar{c}_t}{d\pi_t}\right) = -\frac{1}{1-\bar{\Theta}}\left[\mathbb{E}_I(K^i)\mathbb{E}_I(\Omega^i K^i) + Cov_I(K^i, \Omega^i K^i)\right] \tag{48}$$

The effects on each of the terms is especially clear if we further assume that all households share the same α_1^i, ρ_π^i, and ρ_y^i, and so the same Ω^i. In that case equation 48 becomes:

$$\frac{d}{d\pi_t}\left(\frac{d\bar{c}_t}{d\pi_t}\right) = -\frac{1}{1-\bar{\Theta}}\Omega\left[\left(\mathbb{E}_I(K^i)\right)^2 + Var_I(K^i)\right] \tag{49}$$

The elasticity of aggregate consumption to inflation therefore falls for two reasons as the inflationary shock gets larger. First, the average inflation perception rises, so the average subjective model becomes more negative about inflation. Second, the narrative heterogeneity channel also decreases $d\bar{c}_t/d\pi_t$. As the shock size increases, the difference between the inflation perceptions of attentive (high K^i) and less attentive (low K^i) households grows. The most attentive households therefore adjust their subjective models more towards lower $\hat{\alpha}_t^i$ relative to inattentive households, which makes the covariance of K^i and $\partial c_t^i/\partial\tilde{\mathbb{E}}_t^i\pi_t$ more negative. Intuitively, as the most attentive households adjust their perceptions by the most, larger shocks lead to a greater concentration of very negative subjective models among the most attentive households. This effect is particularly strong if information choices are very heterogeneous across households, as suggested by the evidence in e.g. Link et al. (2021).

History dependence. If households believe inflation is persistent, recent inflation history will also affect the distribution of inflation perceptions. Differentiating equation 47 with respect to realized inflation in period $t-1$ gives:

$$\frac{d}{d\pi_{t-1}}\left(\frac{d\bar{c}_t}{d\pi_t}\right) = -\frac{1}{1-\bar{\Theta}}\left[\mathbb{E}_I(K^i)\mathbb{E}_I(\Omega^i K^i(1-K^i)\rho_\pi^i) + Cov_I(K^i, \Omega^i K^i(1-K^i)\rho_\pi^i)\right] \tag{50}$$

The first effect is as with the size dependence: high inflation in period $t-1$ implies high average inflation perceptions in period t (through higher prior beliefs), which lowers the aggregate response to inflation.

The narrative heterogeneity effect is more subtle. Again assuming that households all share the same Ω^i, equation 50 becomes:

$$\frac{d}{d\pi_{t-1}}\left(\frac{d\bar{c}_t}{d\pi_t}\right) = -\frac{1}{1-\bar{\Theta}}\Omega\rho_\pi\left[\mathbb{E}_I(K^i)\mathbb{E}_I(K^i(1-K^i)) + Cov_I(K^i, K^i(1-K^i))\right] \tag{51}$$

The second term may be positive or negative, because there are two opposing effects: on the one hand, as for the size dependence, any inflation shock has the greatest effects within the period on the perceptions of the most attentive households. This acts to reduce $d\bar{c}_t/d\pi_t$. However, on the other hand, the most attentive households are the least reliant on their prior beliefs when forming perceptions of π_t, and so are least affected by their past inflation perceptions. If average K^i is sufficiently large, this second effect dominates and high past inflation increases the covariance of information and $\partial c_t^i/\partial\tilde{\mathbb{E}}_t^i\pi_t$, and so increases $d\bar{c}_t/d\pi_t$.

5.3 Quantification

To calibrate the model, I proceed in four steps. First, I set some parameters to standard values in the literature. Second, I assume that all households have the same subjective models for the determination of π_t, r_t, and that these coincide with the true laws of motion of those variables. I therefore set the parameters of those laws of motion, and of equations 26 and 27, to match the output from an OLS estimation of those equations, using UK macroeconomic time-series data from 2001-2019. Third, I assume all households hold the same subjective models for y_t except for α^i, and I use a similar estimation of equation 28 to obtain the values of all parameters except α^i. The estimated coefficient on π_t in that equation is used as the mean α^i across households. The coefficients in the true law of motion for y_t are determined as functions of other calibrated variables, detailed in Appendix D.4.

Fourth, I assume that α_0^i is normally distributed across households, and that all households share the same α_1^i. The common subjective model for π_t implies they also share the same δ^{i*}. I choose $\mathrm{Var}_I(\alpha_0^i)$, α_1, and δ^* to target three key moments from the IAS data: the average proportion of households who believe inflation makes the economy weaker, the elasticity of this proportion to increases in inflation, and an estimate of the average Kalman gain in inflation perceptions. I use this last moment rather than the estimated marginal effects in Table 1 as it is not clear how to position the boundaries of the 'makes little difference' survey answer in the model. Full details of the calibration are in Appendix F.

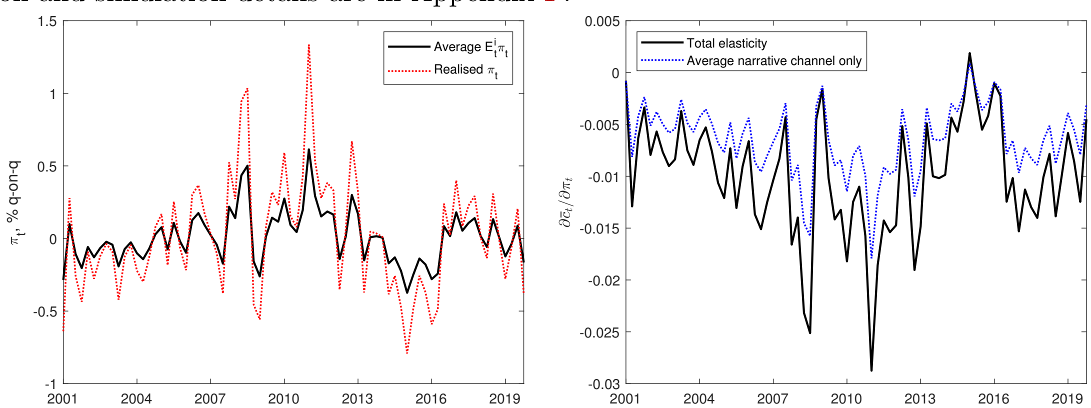
Note that the estimation of equations 26 - 28 used in this calibration, while naïve from the point of view of modern empirical macroeconomics, are not naïve from the households' point of view. In their subjective models, these regressions uncover the intended underlying parameters. In this it is important that y_t does not appear in the law of motion for π_t.

With this calibration, I conduct two exercises. In both, I consider $\partial\bar{c}_t/\partial\pi_t$, the partial-equilibrium response of consumption to an inflation shock holding r_t, y_t constant, as this is the component of shock transmission over which the narrative heterogeneity channel operates.

First, I find the size of the narrative heterogeneity channel for an inflation shock arriving in steady state. I obtain a stationary distribution of inflation perceptions by assuming that $\pi_t = 0$ for many periods, so the only variation in $\tilde{\mathbb{E}}_t^i \pi_t$ comes from idiosyncratic noise in household signals. In the steady state with $\tilde{\mathbb{E}}_t^i \pi_t$ drawn from this distribution, $\partial \bar{c}_t / \partial \pi_t$ is negative, because the majority of households believe inflation weakens the economy in the survey. As observed in the IAS data, there is a negative correlation between information K^i and $\partial c_t^i / \partial \tilde{\mathbb{E}}_t^i \pi_t$, so the narrative heterogeneity channel is also negative. Quantitatively, it reduces steady state $\partial \bar{c}_t / \partial \pi_t$ by 56%.

Second, I simulate the model for 1000000 households, feeding in the path of de-meaned quarterly CPI inflation observed in the UK over the sample period as realizations of π_t. Figure 5 shows the paths of average perceived inflation and $\partial \bar{c}_t / \partial \pi_t$. Compared to realized inflation, average perceived inflation is relatively smooth. However, this still implies substantial volatility in $\partial \bar{c}_t / \partial \pi_t$.

Figure 5: Simulated inflation perceptions and aggregate consumption elasticity to inflation. Calibration and simulation details are in Appendix F.



(a) Average $\tilde{\mathbb{E}}_t^i \pi_t$ and realized π_t (b) $\partial \bar{c}_t / \partial \pi_t$

Using the decomposition from Section 2, we can further split the variation in shock transmission into the representative agent and narrative heterogeneity channels. The blue line in Figure 5b shows $\partial \bar{c}_t / \partial \pi_t$ without the narrative heterogeneity channel. It is substantially less volatile: fluctuations in the covariance of information and subjective models increase the standard deviation of $\partial \bar{c}_t / \partial \pi_t$ by 65%. The reasons for this are explored in Section 5.2.

Note that if there is feedback from demand to inflation, as in standard New Keynesian models, the narrative heterogeneity channel will have further dynamic implications beyond those derived in this paper. The joint distribution of information and subjective models will affect, and be affected by, the distributions of inflation and other variables. The exercises presented here should therefore be viewed as a first step in understanding the wide-ranging implications of this novel channel of shock transmission.

The other restrictive assumption made here is that information is only limited and heterogeneous about inflation, and the subjective model updating only occurs in a single parameter α^i. In reality these features may be common to information on many different variables, and many aspects of subjective models. This means that narrative heterogeneity effects are potentially much more widespread than I allow for here. The full implications of narrative heterogeneity effects among households are therefore likely to be larger, and richer, than those derived here. I focus on these first-round effects, however, as the IAS data cannot discipline the behavior of information on other variables, or other aspects of subjective models.

6 Endogenous long-run expectations

So far in this analysis, information about inflation has mostly affected expectations about aggregate variables in the near future, as all variables are perceived to be stationary. Policymakers, however, are often also concerned about longer-term expectations (e.g. Powell, 2021). In this section I extend the model to allow households to use current information to update their expectations of long-run inflation, and derive implications for aggregate dynamics following a large positive inflation shock.

Suppose that household i's subjective model for inflation includes a long-run mean of inflation $\bar{\pi}_t$ which is not necessarily equal to 0:

$$\pi_t = \rho_\pi^i \pi_{t-1} + (1 - \rho_\pi^i)\bar{\pi}_t + u_{\pi t} \tag{52}$$

To begin with, assume that households make choices as if the long-run mean of inflation is a parameter of their subjective model, disregarding its potential to change over time. This extension of the anticipated utility assumption employed in Section 4.6 greatly simplifies the analysis and allows for analytic results, but is not critical for the mechanisms. I relax it in Appendix E.4, and the qualitative results below continue to hold numerically.

Re-deriving the consumption function with this new subjective model for inflation gives (derivation in Appendix E.1):

$$c_t^i = \frac{1-\beta}{1-\beta\rho_y^i} y_t - \sigma\beta r_t + \frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t}\left(\tilde{\mathbb{E}}_t^i \pi_t + \frac{1-\rho_\pi^i}{\rho_\pi^i(1-\beta)} \tilde{\mathbb{E}}_{t-1}^i \bar{\pi}_t\right) \tag{53}$$

where $\tilde{\mathbb{E}}^i_{t-1}\bar{\pi}_t$ is household i's estimate of $\bar{\pi}_t$ before information processing in period t. This consumption function is as in equation 35, except for the additional term in $\bar{\pi}_t$.

In the previous sections, the precision of household information depended on the constant subjective model parameter α^i_0. Now, I allow the perceived long-run mean of inflation to affect that initial model:

$$\alpha^{i,prior}_t = \alpha^i_0 + \alpha^i_1 \tilde{\mathbb{E}}^i_{t-1}\bar{\pi}_t \tag{54}$$

In this way the model allows us to understand the consequences of a rise in long-term inflation expectations for both information and subjective models.

In the interpretation given in Section 4.6, households expecting inflation to deviate from steady state in the long term similarly expect their subjective model of the effects of inflation to deviate from α^i_0, and therefore take that into account in their information choices. I continue with the same microfoundation here. In the resulting rational inattention problem, optimal information choices are as follows:

Lemma 2 *With the subjective model for inflation as in equation 52, and information costs as specified in Appendix D.5, household i chooses a signal of the form:*

$$s^i_t = \pi_t - \tilde{\mathbb{E}}^i_{t-1}\bar{\pi}_t + \varepsilon^i_t, \quad \varepsilon^i_t \sim N(0, \sigma^2_{\varepsilon it}) \tag{55}$$

where $\sigma^2_{\varepsilon it}$ is as implied in equation 37, with $\partial c^i_t / \partial \tilde{\mathbb{E}}^i_t \pi_t$ computed using $\alpha^{i,prior}_t$.

Proof. Appendix E.2. ■

The household then uses this signal to update their beliefs about current inflation, and also their beliefs about the long-run mean $\bar{\pi}_t$. For that updating they therefore acknowledge that $\bar{\pi}_t$ may in fact change over time. Specifically, they assume that $\bar{\pi}_t$ follows a random walk (as in e.g. Cogley and Sbordone, 2008; Fisher et al., 2021):

$$\bar{\pi}_t = \bar{\pi}_{t-1} + v_t, \quad v_t \sim N(0, \sigma^2_v) \tag{56}$$

With this assumption, we can write the household's forecasting problem in state-space form:

$$\xi_t = F^i \xi_{t-1} + e^i_t \tag{57}$$

$$(s^i_t + \tilde{\mathbb{E}}^i_{t-1}\bar{\pi}_t) = C'\xi_t + \varepsilon^i_t \tag{58}$$

where:

$$\xi_t = \begin{pmatrix} \pi_t \\ \bar{\pi}_t \end{pmatrix}, \quad F^i = \begin{pmatrix} \rho_\pi^i & 1-\rho_\pi^i \\ 0 & 1 \end{pmatrix}, \quad e_t^i = \begin{pmatrix} u_{\pi t} + (1-\rho_\pi^i)v_t \\ v_t \end{pmatrix}, \quad C = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{59}$$

It therefore remains optimal for households to incorporate signals into their perceptions of π_t and $\bar{\pi}_t$ using the Kalman filter:

$$\tilde{\mathbb{E}}_t^i \xi_t = (I - K_t^i C')F^i \tilde{\mathbb{E}}_{t-1}^i \xi_{t-1} + K_t^i s_t^i \tag{60}$$

where K_t^i is a 2×1 vector of gain parameters.

This means that households do not use their signals in the way they expected when they made their information decisions, as they did not anticipate the update to beliefs about $\bar{\pi}_t$. This is a direct consequence of the anticipated utility assumption, relaxed in Appendix E.4. To avoid $K_t^i = 0$ becoming an absorbing state, I further add that each household has a small probability of resetting to $\tilde{\mathbb{E}}_{t-1}^i \bar{\pi}_t = 0$ each period. As with other fluctuations in $\bar{\pi}_t$ beliefs, households do not take this reset shock into account when making information choices.

Proposition 5 shows how perceived long-run inflation affects optimal attention.

Proposition 5 *Let $\sigma_{\varepsilon it}^{2*}$ denote the optimally chosen noise variance in s_t^i. Then, for $\sigma_{\varepsilon it}^{2*} < \infty$:*

$$\frac{\partial \sigma_{\varepsilon it}^{2*}}{\partial \tilde{\mathbb{E}}_{t-1}^i \bar{\pi}_t} < 0 \quad \text{if and only if} \quad \left.\frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t}\right|_{\alpha_t^{i,prior}} < 0 \tag{61}$$

$$\frac{\partial K_t^i}{\partial \tilde{\mathbb{E}}_{t-1}^i \bar{\pi}_t} > 0 \quad \text{if and only if} \quad \left.\frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t}\right|_{\alpha_t^{i,prior}} < 0 \tag{62}$$

Proof. Appendix E.2. ∎

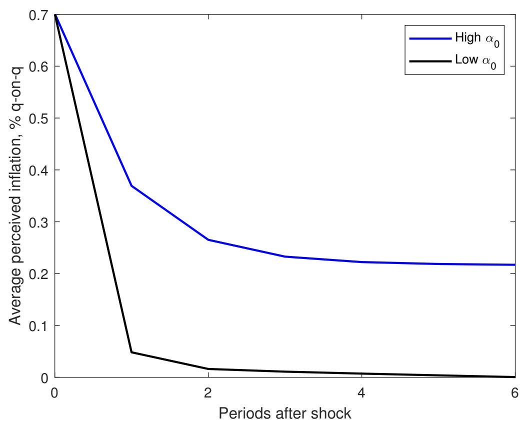
If a household starts the period with a negative subjective model, such that $\partial c_t^i / \partial \tilde{\mathbb{E}}_t^i \pi_t < 0$, then higher long-run inflation expectations cause them to update their subjective model to be even more negative about the effects of inflation (equation 54). This increases the magnitude of their consumption response to inflation, so information becomes more valuable, and they pay to acquire more precise signals. Their perceptions of π_t and $\bar{\pi}_t$ become more responsive to realized π_t as a result.

The reverse is true for a household with $\partial c_t^i / \partial \tilde{\mathbb{E}}_t^i \pi_t > 0$ under $\alpha_t^{i,prior}$. Higher long-run expected inflation similarly reduces their $\alpha_t^{i,prior}$, but that shifts $\partial c_t^i / \partial \tilde{\mathbb{E}}_t^i \pi_t$ towards zero. Inflation is believed to matter less for decisions, which reduces the value of inflation information. Perceived current and long-run inflation get less responsive to realized π_t.

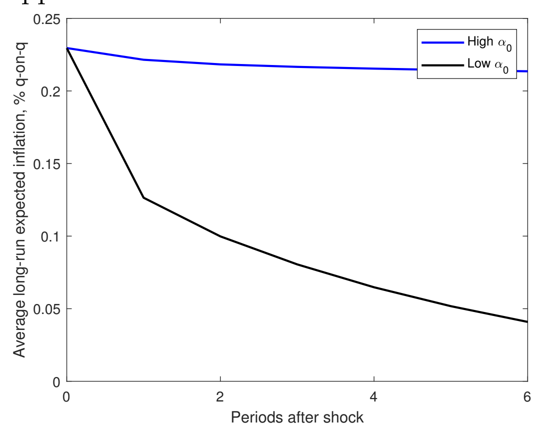
Information about π_t therefore not only affects the subjective model in period t, but also the subjective model used to make information choices in $t+1$, through perceptions of $\bar{\pi}_t$. These interdependencies imply that the expectations of different households may follow very different paths after a shock. To show this, Figure 6 plots the average perceived π_t and $\bar{\pi}_t$ for two groups of households after a 1 percentage point i.i.d. inflation shock. Within a group, all households share the same subjective model parameters, but observe idiosyncratic signals.

The figure is drawn assuming all households have $\tilde{\mathbb{E}}^i_{t-1}\bar{\pi}_t = 0$ when the shock hits, and prior beliefs in the period of the shock are drawn from the stationary distribution obtained in the absence of aggregate shocks.

Figure 6: Simulated average $\tilde{\mathbb{E}}^i_t\pi_t$ and $\tilde{\mathbb{E}}^i_{t-1}\bar{\pi}_t$ for two household groups after an i.i.d. inflation shock. Calibration and simulation details are in Appendix F.



(a) Average $\tilde{\mathbb{E}}^i_t\pi_t$



(b) Average $\tilde{\mathbb{E}}^i_{t-1}\bar{\pi}_t$

The first group of households, shown in black, begin the shock period with low α^i_0, so they have $\partial c^i_t/\partial\tilde{\mathbb{E}}^i_t\pi_t < 0$ and $K^i_t > 0$. Since they process some information, both perceived current and long-run inflation rise when the shock hits. However, as this leads them to increase their information processing, they observe that inflation has fallen in the periods after the shock, and their perceptions quickly return to zero.

The second group of households, shown in blue, are identical to the first except that they have a higher α^i_0, such that $\partial c^i_t/\partial\tilde{\mathbb{E}}^i_t\pi_t > 0$. Their α^i_0 has been chosen such that both groups have the same K^i_t in the period of the shock, so average inflation perceptions initially rise by the same amount. However, the rise in perceived $\bar{\pi}_t$ causes this second group to pay *less* attention to inflation. This slows down the return of long-run expectations, and perceived current inflation, to steady state among this group, as they do not precisely observe the fall in inflation after the shock. In turn, this means their attention remains low.

High inflation can therefore become 'baked in' to expectations, but only among households who start out believing inflation strengthens the economy, and who subsequently reduce their attention after an inflationary shock. This is a novel effect from the interaction of the two components of expectations: if households had limited information but knew the true equilibrium law of motion for inflation, they would know that the shock is transitory, and would not update their long-run expectations. If households didn't know the true model but had full information, they would all observe inflation returning to zero after the shock.

Empirical evidence. As the IAS does not contain a panel dimension, we cannot track individual households over time to test this mechanism directly. However, we can test the underlying process. Proposition 5 implies that among those with negative subjective models of inflation, higher perceived inflation is associated with more information processing.[29] Among those with positive models, that correlation should be reversed. I test this in the survey data in Appendix E.3, and find evidence of the relevant correlations, lending support to the mechanism in the model.

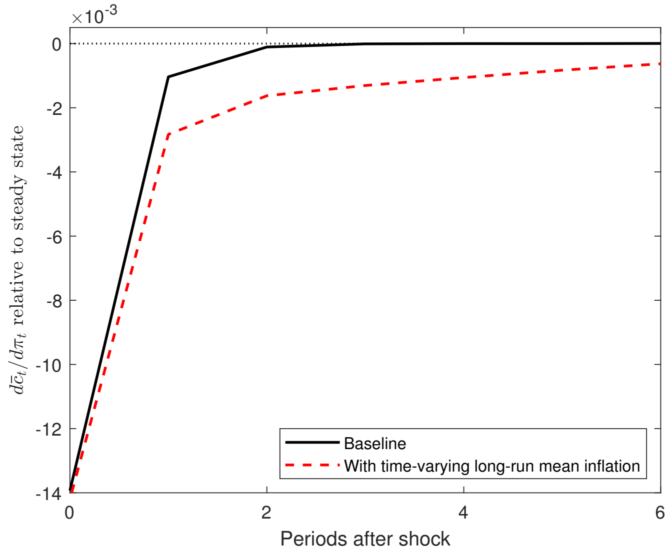
This rationalizes a key result in Pfajfar and Santoro (2010): in the Michigan Survey of Consumers, they estimate that higher inflation is principally associated with more frequent information acquisition among those with higher than average expected inflation. In the model developed here, those households mostly hold negative subjective models of the effects of inflation, and so Proposition 5 generates the result. Similarly, Link et al. (2022) find that greater information acquisition about inflation is associated with higher expected inflation on average, even though this implies greater average forecast errors. Again this is explained by Proposition 5, combined with the observation that most households in the data believe inflation weakens the economy.

Implication for aggregate dynamics. The fact that this 'baking in' is correlated with subjective models implies that it has a persistent effect on the aggregate transmission of inflationary shocks. Figure 7 shows $d\bar{c}_t/d\pi_t$ in the calibrated model (Section 5.3) after the one-off inflationary shock from Figure 6, with and without time-varying long-run perceptions.

The aggregate consumption elasticity to inflation returns quickly to its pre-shock level when long-run expectations remain at 0, because the perceived persistence of inflation in the calibration is low. However, with time-varying perceived long-run inflation, $d\bar{c}_t/d\pi_t$ remains depressed after the shock, because of the households whose expectations have become 'baked in' at a high level. Their subjective models of the effects of inflation are persistently less

[29] As in other surveys, households overestimate inflation on average (Carroll, 2003; Kumar et al., 2015), so this implies households with more information make larger forecast errors.

Figure 7: Simulated aggregate consumption elasticity to inflation, relative to steady state, after an i.i.d. inflation shock. Calibration and simulation details are in Appendix F.



positive than before the shock.

This has an effect on the average subjective model, but also importantly on the covariance of information and subjective models. A group of well-informed households who believed in positive effects of inflation move to being uninformed, which persistently lowers the narrative heterogeneity channel. Intuitively, inflation information becomes more concentrated among those who react to it in the most negative way. Decomposing the changes in $d\bar{c}_t/d\pi_t$ reveals that the narrative heterogeneity channel accounts for 70% of the difference between $d\bar{c}_t/d\pi_t$ and its pre-shock value after 6 quarters.

7 Conclusion

This paper studies the transmission of aggregate shocks through heterogeneity in expectation formation. Importantly, it allows for interactions between the information and subjective models involved in forming expectations, which previous literature has treated separately.

In a general log-linear model, shocks pass through to aggregate actions along three channels. The first is the transmission that would be seen in a representative agent model. The second comes from heterogeneity in the parameters of policy functions, extending well-known results from the literature on heterogeneous-agent macroeconomics. The third channel is novel. The narrative heterogeneity channel operates when information and subjective models covary systematically across agents. Heterogeneous subjective models imply heterogeneous responses to information, so systematic patterns in the distribution of information across agents with different subjective models distort the aggregate response to shocks.

I use unique features of the Bank of England Inflation Attitudes Survey to document that subjective models and information about inflation do indeed covary systematically with each other, and with inflation perceptions and expectations. The distribution of subjective models also varies systematically with realized inflation. In a model matching these patterns the narrative heterogeneity channel substantially reduces the aggregate consumption response to inflationary shocks, generates size- and history-dependence in that transmission, and implies that temporarily high inflation may become 'baked in' to certain household expectations.

When tracking if high inflation is becoming 'baked in' to expectations, not all households are therefore of equal concern. The households who believed before the shock that more inflation would make the economy stronger pose the greatest risk, because they reduce their attention to inflation as perceived inflation rises. If their expectations increase substantially, reducing realized inflation will not be sufficient to bring their expectations back down. Over the 12 months to August 2022, the perceived inflation of households in this positive group in the IAS rose by just 1.5 percentage points, substantially less than the average rise in perceived inflation across all households in the survey (4.1 %pts). This suggests that the cat was not yet out of the bag in UK inflation expectations in the summer of 2022.

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A Log-linear model proofs and derivations

A.1 Proofs of Propositions 1 - 3

Proposition 1. The derivative of the expectation of each element z^i_{jt} of $\boldsymbol{z}^i_t$ can be decomposed using the chain rule:

$$\frac{d\mathbb{E}^i_t z^i_{jt}}{d\xi_t} = \left.\frac{d\mathbb{E}^i_t z^i_{jt}}{d\xi_t}\right|_{\mathbb{E}^i_t z_{k\neq j,t}} + \sum_{k\neq j}^{N_z} \frac{\partial \mathbb{E}^i_t z^i_{jt}}{\partial \mathbb{E}^i_t z^i_{kt}} \frac{d\mathbb{E}^i_t z^i_{kt}}{d\xi_t} \tag{63}$$

Stacking this expression over all elements of $\boldsymbol{z}^i_t$ and rearranging gives:

$$\frac{d\mathbb{E}^i_t \boldsymbol{z}^i_t}{d\xi_t} = (\boldsymbol{I} - \boldsymbol{\mathcal{M}}^i_t)^{-1} \boldsymbol{\delta}^i_t \tag{64}$$

which substituted into equation 2 gives the result.

Proposition 2. From the definition of $\bar{x}_{st}$ (equation 12), we have:

$$\frac{d\bar{x}_{st}}{d\xi_t} = \mathbb{E}_I \frac{dx^i_{st}}{d\xi_t} \tag{65}$$

The s^{th} row of equation 3 can be written as:

$$\frac{dx^i_{st}}{d\xi_t} = \sum_{j=1}^{N_z} \sum_{k=1}^{N_z} \mu^i_{sj,t} \chi^i_{jk,t} \delta^i_{k,t} \tag{66}$$

Substituting this into equation 65 gives:

$$\frac{d\bar{x}_{st}}{d\xi_t} = \sum_{j=1}^{N_z} \sum_{k=1}^{N_z} \mathbb{E}_I \mu^i_{sj,t} \chi^i_{jk,t} \delta^i_{k,t} \tag{67}$$

From the definition of covariance, $\mathbb{E}(XY) = \mathbb{E}(X)\mathbb{E}(Y) + Cov(X, Y)$ for any X, Y. Applying this to equation 67 implies:

$$\frac{d\bar{x}_{st}}{d\xi_t} = \sum_{j=1}^{N_z} \sum_{k=1}^{N_z} \left[\bar{\mu}_{sj,t} \mathbb{E}_I(\chi^i_{jk,t} \delta^i_{k,t}) + Cov_I(\mu^i_{sj,t}, \chi^i_{jk,t} \delta^i_{k,t}) \right] \tag{68}$$

Applying the covariance formula again to the first term inside the sum in equation 68 implies equation 13.

Proposition 3. Differentiating equation 14 with respect to ξ_t we have:

$$A \frac{d\boldsymbol{z}_t}{d\xi_t} + B \frac{d\bar{\boldsymbol{x}}_t}{d\xi_t} + Ce_\xi = 0 \tag{69}$$

From Proposition 1 and Assumption 2 we have:

$$\frac{d\bar{\boldsymbol{x}}_t}{d\xi_t} = \mathbb{E}_I \left(\boldsymbol{\mu}^i_t \boldsymbol{\chi}^i_t \tilde{\boldsymbol{\delta}}^i_t \right) \frac{d\boldsymbol{z}_t}{d\xi_t} \tag{70}$$

Substituting equation 70 into equation 69 and rearranging:

$$\frac{d\boldsymbol{z}_t}{d\xi_t} = -\left(A + B\mathbb{E}_I \left(\boldsymbol{\mu}^i_t \boldsymbol{\chi}^i_t \tilde{\boldsymbol{\delta}}^i_t \right) \right)^{-1} Ce_\xi \tag{71}$$

Substituting equation 71 into equation 70 yields equation 16.

A.2 Consumption function in a standard household problem

Household i maximizes:[30]

$$\mathbb{E}^i_t \sum_{s=0}^{\infty} \beta^s \frac{(C^i_{t+s})^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \text{ s.t. } C^i_{t+s} + B^i_{t+s} = \tilde{R}_{t+s-1} B^i_{t+s-1} + Y_{t+s} \tag{72}$$

where C^i_t is consumption, σ is the intertemporal elasticity of substitution, B^i_t are real one-period bonds bought in period t, $\tilde{R}_t$ is the gross real interest rate on such a bond, and Y_t is real income (assumed equal across households). The first order condition is the standard

[30]This derivation closely follows that in Bilbiie (2019) appendix A, and is also similar to consumption functions derived in Farhi and Werning (2019) and others.

Euler equation. Log-linearizing about steady state and substituting forward we obtain:

$$c_t^i = \mathbb{E}_t^i c_{t+s}^i - \sigma \sum_{k=0}^{s-1} \mathbb{E}_t^i \tilde{r}_{t+k} \tag{73}$$

where lower-case letters denote log-deviations from steady state. Assuming that $b_t^i = 0$ (as it is in equilibrium in a standard representative-agent or two-agent New Keynesian model), the log-linearized present value budget constraint is:

$$\sum_{s=0}^{\infty} \beta^s \mathbb{E}_t^i (c_{t+s}^i - \sum_{k=0}^{s-1} \tilde{r}_{t+k}) = \sum_{s=0}^{\infty} \beta^s \mathbb{E}_t^i (y_{t+s} - \sum_{k=0}^{s-1} \tilde{r}_{t+k}) \tag{74}$$

Use the Euler equation to substitute out for $\mathbb{E}_t^i c_{t+s}^i$ to obtain:

$$\sum_{s=0}^{\infty} \beta^s (c_t^i - (1-\sigma)\mathbb{E}_t^i \sum_{k=0}^{s-1} \tilde{r}_{t+k}) = \sum_{s=0}^{\infty} \beta^s \mathbb{E}_t^i (y_{t+s} - \sum_{k=0}^{s-1} \tilde{r}_{t+k}) \tag{75}$$

Rearranging:

$$\frac{1}{1-\beta} c_t^i = \sum_{s=0}^{\infty} \beta^s \mathbb{E}_t^i y_{t+s} - \frac{\sigma\beta}{1-\beta} \sum_{s=0}^{\infty} \beta^s \mathbb{E}_t^i \tilde{r}_{t+s} \tag{76}$$

Multiplying through by $1-\beta$, and applying the Fisher equation $\mathbb{E}_t^i \tilde{r}_t = \mathbb{E}_t^i (r_t - \pi_{t+1})$ (where r_t is the nominal interest rate), we obtain equation 6.

B Defining the direct information indicator in the IAS

The full set of questions used to construct the information dummy is set out below, along with the dates at which each was asked and how the answers are mapped into the information indicator used above. Note that my question numbering differs from the labels in the IAS microdata, to aid the logical organization of the paper. All of the questions were only asked in the first quarter of the year(s) indicated. In the main exercises I exclude questions 2e and 2g from the total information variable, to ensure that there are no periods in which two questions are asked. I remove these rather than the short run questions in those periods to keep the majority of questions as short run expectations. The results are robust to including these extra questions. See Appendix C.2 for this, and robustness checks with other variations in the definition of the information indicator.

Question 2b *What were the most important factors that led you [to change (insert their response to how expectation has changed)] your expectation of prices in the shops over the next 12 months?*

Please select up to 4:

1. *How prices have changed in the shops recently, over the last 12 months*
2. *How prices have changed in the shops, on average, over the longer term i.e the last few years*
3. *Reports of current inflation in the media*
4. *Discussion of the prospects for inflation in the media*
5. *The level of interest rates*
6. *The inflation target set by the government*
7. *The current strength of the UK economy*
8. *Expectations about how economic conditions in the UK are likely to evolve*
9. *The level of the exchange rate (the value of sterling)*
10. *Other factors*
11. *None*

Asked: 2017
Information indicator: =1 if items 3 or 4 selected, =0 otherwise.

Question 2c *What were the most important factors that led you to change/not change your expectation of prices in the shops in the longer term?*

1. *How prices have changed in the shops recently, over the last 12 months*
2. *How prices have changed in the shops, on average, over the longer term i.e the last few years*
3. *Reports of current inflation in the media*
4. *Discussion of the prospects for inflation in the media*
5. *The level of interest rates*
6. *The inflation target set by the government*
7. *The current strength of the UK economy*
8. *Expectations about how economic conditions in the UK are likely to evolve*
9. *The level of the exchange rate (the value of sterling)*
10. *Other factors*
11. *None*

Asked: 2018, 2019
Information indicator: =1 if items 3 or 4 selected, =0 otherwise.

Question 2d *When you said prices would go up in the next 12 months, how important were the following things in getting to that answer?*

For each option, possible answers are:

- *Very important*
- *Fairly important*
- *Not very important*
- *Not at all important*
- *Don't know*
- *Refused*

Options:

1. *How prices have changed in the shops in your most recent visits (i.e. the last 1 to 6 months).*
2. *How prices have changed in the shops over the longer term (i.e. the last 12 months or more)*
3. *The current level of interest rates.*
4. *The current strength of the British Economy.*
5. *The inflation target set by the government.*
6. *Reports on inflation outlook in the media.*
7. *Reports of VAT changes in the media.*
8. *Other factor(s).*

Asked: 2009, 2010, 2011, 2013
Information indicator: =1 if 'very important' selected for option 6, =0 otherwise.

Question 2e *And which, if any, of the same factors were important in getting to your expectation of how prices will change over the longer term (say in 5 years time)?*

1. *How prices have changed in the shops in your most recent visits (i.e. the last 1 to 6 months).*
2. *How prices have changed in the shops over the longer term (i.e. the last 12 months or more)*
3. *The current level of interest rates.*
4. *The current strength of the British Economy.*
5. *The inflation target set by the government.*
6. *Reports on inflation outlook in the media.*
7. *Reports of VAT changes in the media.*
8. *Other factor(s).*

Asked: 2011, immediately after Question 2d
Information indicator: =1 if item 6 selected, =0 otherwise.

Question 2f *What were the most important factors in getting to your expectation for how prices in the shops would change over the next 12 months?*

Please select up to 4:

1. *How prices have changed in the shops recently, over the last 12 months*
2. *How prices have changed in the shops, on average, over the longer term i.e the last few years*
3. *Reports of current inflation in the media*
4. *Discussion of the prospects for inflation in the media*
5. *The level of interest rates*
6. *The inflation target set by the government*
7. *The current strength of the UK economy*
8. *Expectations about how economic conditions in the UK are likely to evolve*
9. *Other factors*
10. *None*

Asked: 2016
Information indicator: =1 if items 3 or 4 selected, =0 otherwise.

Question 2g *And what were the most important factors in getting to your expectation for how prices in the shops would change over the longer term (say in 5 years' time)?*

Please select up to 4:

1. *How prices have changed in the shops recently, over the last 12 months*
2. *How prices have changed in the shops, on average, over the longer term i.e the last few years*
3. *Reports of current inflation in the media*
4. *Discussion of the prospects for inflation in the media*
5. *The level of interest rates*
6. *The inflation target set by the government*
7. *The current strength of the UK economy*
8. *Expectations about how economic conditions in the UK are likely to evolve*
9. *Other factors*
10. *None*

Asked: 2016
Information indicator: =1 if items 3 or 4 selected, =0 otherwise.

C Further empirical results

C.1 The relationship of planned consumption with measured information and subjective models

To confirm that the survey measures of information and subjective models uncover meaningful aspects of household beliefs, I consider how they correlate with planned consumption behavior. To this end, I use the following survey question:

Question 3 *Which, if any, of the following actions are you taking, or planning to take, in the light of your expectations of price changes over the next twelve months?*

- *Cut back spending and save more.*

Crucially, this asks about consumption choices which are explicitly driven by expected inflation.[31] A household answering 'yes' to this question, and who reports elsewhere in the survey that they expect prices to rise in the next year, is therefore indicating that $dc_t^i/d\mathbb{E}_t^i p_{t+1} < 0$. A question that only asked about consumption or consumption changes, without reference to the cause of the behavior, would conflate this with reactions to expectations of other variables, which might also be influenced by the same shocks as expected inflation, either directly or through cross-learning. Question 3 is therefore informative about the sign of $\frac{dc_t^i}{d\mathbb{E}_t^i p_{t+1}}$. If current prices are taken as given by the household, then this is the same as the sign of $\frac{dc_t^i}{d\mathbb{E}_t^i \pi_{t+1}}$.

The vast majority of respondents (98%) expect positive inflation over the next 12 months.[32]

[31] Another question in the survey asks if the respondent will "bring forward major purchases such as furniture or electrical goods" as a result of expected inflation. I do not use this for two reasons. First, as Nunes and Park (2020) note, the question refers specifically to durable goods, which may not respond to prices in the same way as aggregate consumption, the object of interest. Second, it is very rarely chosen: just 6% of respondents said they would bring forward major purchases. In contrast, 40% report that they will cut back spending and save more. Any estimation on this variable will therefore be heavily influenced by a small subset of agents.

[32] The analysis in this section excludes any households who report expecting zero inflation over the next 12 months, or who do not answer the inflation expectation question, as Question 3 is difficult to interpret for these households. I discuss the appropriate counterfactual implicit in the question below. Including these people, 79% of respondents to Question 3 expect positive inflation, 7% expect zero inflation, 2% expect deflation, and 12% do not answer.

For these households, yes and no responses to Question 3 respectively indicate that:

$$\frac{dc_t^i}{d\mathbb{E}_t^i p_{t+1}} \begin{cases} < 0 & \text{if answer yes} \\ \geq 0 & \text{if answer no} \end{cases} \tag{77}$$

For the minority who expect deflation, these inequalities are reversed: responding with 'yes' indicates consumption is being cut because of an expected fall in prices. I therefore define the following indicator:

$$\frac{\widetilde{dc_t^i}}{d\mathbb{E}_t^i p_{t+1}} = \begin{cases} 1 & \text{if Q3='no' and } \mathbb{E}_t^i \pi_{t+1} > 0 \\ 0 & \text{if Q3='yes' and } \mathbb{E}_t^i \pi_{t+1} > 0 \\ 1 & \text{if Q3='yes' and } \mathbb{E}_t^i \pi_{t+1} < 0 \\ 0 & \text{if Q3='no' and } \mathbb{E}_t^i \pi_{t+1} < 0 \end{cases} \tag{78}$$

For the large majority who expect inflation, this is equal to 1 if $\frac{dc_t^i}{d\mathbb{E}_t^i p_{t+1}} \geq 0$, and equal to 0 if the reaction to expected price rises is strictly negative. The same is true of the minority who expect deflation, except that any household with $\frac{dc_t^i}{d\mathbb{E}_t^i p_{t+1}} = 0$ would respond 'no' to Question 3, and so is counted as if their response to expected price rises is strictly negative. The mislabeling is not a large issue, as less than 1% of respondents to Question 3 both expect deflation and answer 'no'. The results below are robust to removing the few households who expect deflation (see Table 3 column 2).

Table 3 shows how this is related to the information indicator and the subjective models (responses to Question 1). Column 1 shows the results from estimating a probit regression of $\frac{\widetilde{dc_t^i}}{d\mathbb{E}_{+1}}$ on the information indicator interacted with subjective models (Question 1), plus the standard household controls and time fixed effects used above. The coefficient on information is significantly negative for those with negative subjective models of inflation, despite the fact that substitution effects imply $\frac{dc_t^i}{d\mathbb{E}_t^i p_{t+1}} \geq 0$ in many standard models. Being informed is therefore associated with a *lower* probability of responding positively to expected inflation for these households.

However, for those who believe inflation makes the economy stronger, being informed is associated with a significantly higher $\Pr(\frac{dc_t^i}{d\mathbb{E}_t^i \pi_{t+1}} \geq 0)$. For those who believe inflation makes no difference, the average value of $\Pr(\frac{dc_t^i}{d\mathbb{E}_t^i \pi_{t+1}} \geq 0)$ with and without information, which is also consistent with the interpretation of these variables as $\frac{\widetilde{dc_t^i}}{d\mathbb{E}_{+1}} = 1$ includes the case where $\frac{dc_t^i}{d\mathbb{E}_t^i \pi_{t+1}} = 0$.

Table 3: Consumption response to inflation correlates with information, by subjective model

	(1) c response to Eπ	(2) c response to Eπ
information indicator=1	-0.213***	-0.224***
	(0.0611)	(0.0613)
end up stronger	0.0108	0.0392
	(0.0891)	(0.0906)
information indicator=1 × end up stronger	0.348*	0.313*
	(0.185)	(0.186)
make little difference	0.130**	0.157***
	(0.0594)	(0.0600)
information indicator=1 × make little difference	0.0240	-0.0149
	(0.126)	(0.128)
dont know	0.0958	0.0978
	(0.0833)	(0.0846)
information indicator=1 × dont know	-0.0158	-0.0342
	(0.186)	(0.187)
Expected Inflation	All	Exclude Deflation
Controls	All	All
Time FE	Yes	Yes
Observations	4940	4871

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Note: The table reports the results of probit regressions of the $\frac{\widetilde{dc_t^i}}{d\mathbb{E}_t^i \pi_{t+1}}$ indicator on the information indicator, interacted with responses to Question 1. The omitted category is a household with information indicator=0 who holds the belief that inflation makes the economy weaker. All regressions are weighted using the survey weights provided in the IAS.

This is consistent with individuals filtering information through their subjective models of the economy. If a household who believes inflation weakens the economy gets more information about future positive inflation, their subjective model implies that they should cut consumption, because bad times lie ahead. If instead a household believes inflation strengthens the economy, then they will react in the opposite way to the same inflation. The overall correlation of information and consumption response is negative because the majority of households believe inflation makes the economy weaker. This therefore supports the claim that the information indicator and answers to Question 1 reflect the information and subjective models used by households in making their consumption decisions.

The analysis here assumes that when asked whether they will cut back consumption and save more, households are comparing their actions to a counterfactual in which there are no price rises over the next 12 months. An alternative possibility is that they are comparing with a consumption plan made in the past, in which case the relevant counterfactual is where expected inflation is unchanged from the level expected when the plan was made. I consider this in two ways, and find that the qualitative patterns in reported consumption responses to inflation are the same for households expecting inflation to increase or decrease relative to the previous year. It does not therefore appear that past inflation is the relevant counterfactual for most respondents.

First, column 2 of Table 3 re-runs the regression in column 1, excluding any respondent who reports expecting prices to fall over the next year. All results are qualitatively the same as over the full sample, showing that the few respondents expecting deflation are not driving the results.

Second, I split the sample by the sign of the respondent's expected change in inflation, computed as the sign of the difference between 12-month ahead inflation forecast and their perception of inflation over the previous 12 months. The results are in Table 4. The sample sizes in each group are substantially smaller than over the full sample, so some significance is lost, but importantly the signs of the key coefficients remain the same. In each group, households who believe inflation makes the economy weaker are less likely to have $\frac{dc_t^i}{d\mathbb{E}_t^i\pi_{t+1}} \geq 0$ when they get inflation information. For households who believe inflation makes the economy stronger, this effect is reversed. The similarity of these patterns suggests that most respondents use 'no price change' as the counterfactual when answering Question 3, not 'no inflation change'. If the latter was used, we would expect to see changes of sign across the columns in Table 4, as a household expecting a fall in inflation would be reporting $-1 \times \frac{dc_t^i}{d\mathbb{E}_t^i\pi_{t+1}}$, while one expecting a rise in inflation would report $\frac{dc_t^i}{d\mathbb{E}_t^i\pi_{t+1}}$.

Table 4: Consumption response to inflation correlates with information, by subjective model and sign of perceived Eπ change.

	(1) $\text{E}\Delta\pi < 0$	(2) $\text{E}\Delta\pi = 0$	(3) $\text{E}\Delta\pi > 0$
Dc_Dpi			
Information=1	-0.140	-0.305***	-0.257**
	(0.116)	(0.101)	(0.107)
end up stronger	0.0668	-0.178	0.195
	(0.164)	(0.151)	(0.165)
Information=1	0.586	0.349	0.397
× end up stronger	(0.441)	(0.293)	(0.307)
make little	0.165	0.136	0.181
difference	(0.111)	(0.0957)	(0.112)
Information=1	0.129	-0.300	0.113
× make little difference	(0.241)	(0.211)	(0.216)
dont know	0.156	0.0293	0.0264
	(0.176)	(0.128)	(0.167)
Information=1	-0.141	0.469	0.117
× dont know	(0.354)	(0.359)	(0.325)
Controls	All	All	All
Time FE	Yes	Yes	Yes
Observations	1384	1876	1463

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Note: The table reports the results of probit regressions of the $\frac{\widetilde{dc_t^i}}{d\mathbb{E}_t^i \pi_{t+1}}$ indicator on the information indicator, interacted with responses to Question 1, split by the sign of the respondent's inflation expectations. The omitted category in all cases is a household with information indicator=0 who holds the belief that inflation makes the economy weaker. All regressions are weighted using the survey weights provided in the IAS.

C.2 Cross-sectional patterns in information on inflation

Columns 1-3 of Table 5 show the results of probit regressions of the information indicator on subjective models, controls, and period fixed-effects, for three subsamples. The first only uses questions about the information used to arrive at the respondent's *change* in expected inflation, and the second uses only questions about information used to form point forecasts. The third column excludes questions relating to forecast horizons longer than 12 months.

The signs of the marginal effects are the same as in the main exercise in Table 1, though they are not significant in the case of the revisions questions, as the sample size is small.

Table 5: Information correlates with subjective models, split by information question type

	(1) Revision	(2) Point	(3) Short horz.	(4) Extra Qs	(5) Q2d wider	(6) +Other
end up stronger	0.0575	-0.0335	-0.0123	0.00114	-0.00126	-0.00779
	(0.0380)	(0.0218)	(0.0206)	(0.0196)	(0.0196)	(0.0205)
make little difference	-0.0191	-0.0331**	-0.0392***	-0.0310**	-0.0312**	-0.0429***
	(0.0233)	(0.0155)	(0.0141)	(0.0132)	(0.0131)	(0.0139)
dont know	-0.0408	-0.0715***	-0.0622***	-0.0663***	-0.0472***	-0.0663***
	(0.0297)	(0.0206)	(0.0192)	(0.0174)	(0.0180)	(0.0191)
Controls	All	All	All	All	All	All
Time FE	Yes	Yes	Yes	Yes	Yes	Yes
Observations	2364	5906	6848	8306	8270	8270

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Note: The table reports the average marginal effects from estimating probit regressions of the information indicators constructed from subsets of the questions listed in Appendix B on the responses to Question 1. The omitted category is the belief that inflation makes the economy weaker. All regressions are weighted using the survey weights provided in the IAS.

The remaining columns of Table 5 repeat the regression for broader definitions of the information dummy than that used in Table 1. In the fourth column, the information indicator includes Questions 2e and 2g. In the fifth column, I extend the criteria for setting the information indicator equal to 1 in Question 2d to account for the fact that some people may be unwilling to select the highest importance box for any information source. I therefore set the information indicator to 1 if in answer to Question 2d, the respondent selects 'very important' for direct inflation information (as before), or if they do not select 'very important' for any option, but do respond that four or fewer options were 'fairly important', and direct inflation information is among them. In the final column, I set the information indicator =1 if the household chooses a direct information source or 'Other', in case this includes direct information sources (e.g. checking the Bank of England published forecasts). In all of these, the results are robust.

To account for possible selection bias from missing observations, I estimate a version of Table 1 amended for selection as in Heckman (1979). As in Michelacci and Paciello (2020),

Table 6: Information correlates with subjective models, with selection correction

	(1) Information	(2) Information
end up stronger	-0.0178	-0.0183
	(0.0172)	(0.0172)
make little difference	-0.0306**	-0.0315***
	(0.0120)	(0.0121)
dont know	-0.0575***	-0.0579***
	(0.0172)	(0.0173)
Inverse Mills ratio	-0.282***	-0.0696**
	(0.0820)	(0.0355)
Selection stage		
Economic Literacy	0.205***	
	(0.0226)	
HH does not know past π		-0.876***
		(0.0365)
r affects π in 1-2 months		0.0334
		(0.0236)
r affects π in 1-2 yrs		0.0882***
		(0.0231)
Pseudo-R^2 (selection)	0.103	0.127
Controls	All	All
Time FE	Yes	Yes
Observations	18026	18026

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Note: The table reports the coefficients from estimating a linear regression of the information indicator defined in Section 3.1 on the responses to Question 1, augmented with the inverse Mills ratio from a first-stage probit regression of whether the information indicator is observed on measures of economic literacy defined above. The omitted category is the belief that inflation makes the economy weaker. The selection stage is only run for quarters in which the information questions were asked. The model is run using the 2-step limited information method in Heckman (1979). Time fixed effects and controls as in footnote 17 are included in both stages.

I predict observing the relevant survey response using a measure of economic literacy. Here the relevant response is only the information indicator, as there are no missing values for Question 1. Following Michelacci and Paciello (2020), economic literacy is measured with three indicators: the household reports a value for perceived current inflation, and they answer 'agree' or 'strongly agree' to the statements "a rise in interest rates makes prices in the high street rise more slowly in the short term (say a month or two)" and "a rise in interest rates makes prices rise more slowly in the medium term (say a year or two)". I estimate versions of the model with this as an aggregate index (=1 if and only if the household scores on all components), and with the components disaggregated. The results are in Table 6. The predictors used in the first stage are strongly significant. Qualitatively the second-stage results are unchanged from Table 1, and the quantitative differences are small.

C.3 Time series patterns in subjective models of inflation

Bhandari et al. (2019) also study the time series of responses to Question 1, and conclude that households are more pessimistic about inflation when output growth is low. To explore this, I regress the proportion of households responding 'end up weaker' on realized annual CPI inflation and quarterly GDP growth. The results are in column 2 of Table 7. Consistent with Bhandari et al. (2019), the coefficient on GDP growth is significantly negative. However, the R^2 is only slightly higher than that of a regression on inflation only (column 1), so GDP growth does not account for much of the variation in survey answers. Indeed, GDP growth does not have any significant relationship with the proportion of households with a negative view of inflation outside of the four worst months of the Great Recession (column 3).

Table 7: Regressions of the proportion of households answering weaker to Question 1 on aggregate variables.

	(1) Proportion weaker	(2) Proportion weaker	(3) Proportion weaker
Inflation	0.0568***	0.0517***	0.0501***
	(0.00489)	(0.00479)	(0.00469)
GDP growth		-0.0261***	-0.0110
		(0.00869)	(0.0180)
Constant	0.466***	0.487***	0.482***
	(0.0109)	(0.0123)	(0.0152)
Omitted quarters	None	None	2008Q2-2009Q1
R-squared	0.615	0.647	0.554
Observations	70	70	66

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Note: The table reports the results of regressing the proportion of households answering Question 1 that inflation makes the economy weaker on annual CPI inflation and quarter-on-quarter real GDP growth. Proportions are computed using survey weights.

To explore which measure of inflation affects subjective models, Table 8 reports the results of regressing an indicator variable for if the respondent reports a negative subjective model on a variety of inflation measures. The first column uses CPI inflation, so is very similar to the time-series regression in Table 7. Columns 2-4 use more granular measures of the inflation rate experienced by different households, split by whether they are above retirement age (65), above median income, and by their housing tenure. Inflation rates split by these characteristics are provided by the ONS.[33] Column 5 uses perceived current inflation. The different realized inflation measures are strongly correlated, so cannot be included jointly. Although the coefficient sizes vary as the different inflation rates have different levels of volatility, in all cases higher inflation is associated with a significantly greater probability of reporting a negative subjective model. The R^2 is highest for perceived inflation, supporting the choice of modeling assumption in Section **??**.

Table 8: Probability of reporting negative subjective model by experienced and perceived inflation

	(1) Weaker	(2) Weaker	(3) Weaker	(4) Weaker	(5) Weaker
Inflation	0.0510***	0.0463***	0.0457***	0.0292***	0.0254***
	(0.00177)	(0.00170)	(0.00165)	(0.00137)	(0.000720)
Inflation measure	CPI	by retirement	by income	by housing	perceived
Controls	All	All	All	All	All
R-squared	0.0303	0.0286	0.0292	0.0237	0.0371
Observations	68269	68269	68269	68269	68269

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Note: The table reports the results of estimating a linear probability model of whether a respondent reports that inflation makes the economy weaker in response to Question 1 on various measures of inflation. These are annual CPI inflation, inflation split by whether the respondent is of retirement age, split by whether the respondent has above or below median income, split by the respondent's housing tenure, and finally the respondent's perceived current inflation. Sample begins in 2006 Q1, as this is when the ONS sub-group inflation data is available from. Households not reporting a perceived rate of inflation are dropped in all regressions. All regressions are weighted using the survey weights provided in the IAS.

Similar patterns in reverse are observed for the other answers. Table 9 repeats the regressions of Table 7, replacing the dependent variable with the proportion of respondents choosing each of the other answers to Question 1. In all cases, inflation accounts for a large share of the variation in survey answers, and higher inflation is associated with significantly

[33]Finer decompositions of inflation by household characteristics are not reliable, given the data available for the UK (see e.g. Dawber et al., 2022).

lower proportions giving each answer. Higher GDP growth is associated with higher proportions on these other answers, but that relationship is not significantly different from zero for any answer when excluding the worst of the Great Recession.

Table 9: Regressions of the proportion of households giving each answer to Question 1 on aggregate variables.

	(1) Proportion	(2) Proportion	(3) Proportion
Stronger			
Inflation	-0.0123***	-0.0116***	-0.0108***
	(0.00193)	(0.00215)	(0.00221)
GDP growth		0.00346	-0.00392
		(0.00363)	(0.00646)
Constant	0.104***	0.102***	0.104***
	(0.00431)	(0.00550)	(0.00638)
No difference			
Inflation	-0.0292***	-0.0262***	-0.0257***
	(0.00303)	(0.00313)	(0.00314)
GDP growth		0.0150***	0.0106
		(0.00473)	(0.0107)
Constant	0.277***	0.264***	0.266***
	(0.00772)	(0.00883)	(0.0103)
Don't know			
Inflation	-0.0154***	-0.0139***	-0.0135***
	(0.00249)	(0.00262)	(0.00267)
GDP growth		0.00762*	0.00428
		(0.00423)	(0.00987)
Constant	0.153***	0.147***	0.148***
	(0.00687)	(0.00757)	(0.00884)
Omitted quarters	None	None	2008Q2-2009Q1
Observations	70	70	66

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Note: The table reports the results of regressing the proportion of households giving each answer to Question 1 (except 'weaker') on annual CPI inflation and quarter-on-quarter real GDP growth. Proportions are computed using survey weights. The R^2 of the core regressions in column 1 are 0.388 (stronger), 0.534 (no difference), and 0.355 (don't know).

To test if the distribution of beliefs about inflation shifts when the economy reaches the Zero Lower Bound, I estimate an ordered probit regression of subjective models of inflation in the zero lower bound period, and a variety of controls.[34] A response that inflation makes the economy stronger is coded as the highest value, and inflation makes the economy weaker is the lowest value (I exclude the 'don't know' answers). A positive coefficient on the zero lower bound period would therefore imply a shift towards believing inflation makes the economy stronger, as we would expect if households follow a standard New Keynesian model. This is not what the results in Table 10 show: there is no significant shift towards a positive view of inflation in the ZLB period.

Table 10: Ordered probit regressions of subjective models of inflation on whether the economy is at the zero lower bound on nominal interest rates.

	(1) Subjective model	(2) Subjective model	(3) Subjective model
Subjective model			
ZLB	-0.00801	-0.00785	-0.00513
	(0.00937)	(0.00962)	(0.00972)
Controls	None	Household	Household + macro
Observations	83526	83526	83526

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Note: The table reports the results of an ordered probit regression of answers to Question 1 on an indicator for whether the UK economy was at the zero lower bound, defined as the period from 2009Q2 to the end of 2019 (end of the sample). The ordering is: "stronger", "no difference", "weaker". Those answering "no idea" are omitted. All regressions are weighted using the survey weights provided in the IAS.

C.4 Perceived and expected inflation across households

To account for possible selection bias from missing observations, I estimate a version of Table 2 amended for selection as in Heckman (1979). As in Appendix C.2, I predict observing perceived and expected inflation using the components of the economic literacy indicator in Michelacci and Paciello (2020), this time removing the component concerning whether perceived inflation is reported as this is closely related to the dependent variables of the regressions. The results are in Table 11. The predictors used in the first stage are strongly significant. Qualitatively the second-stage results are unchanged from Table 2, and the quantitative differences are small.

[34] The first column of Table 10 has no controls, the second includes the set of household-level covariates used throughout the paper, and the third adds inflation and GDP growth.

Table 11: Perceived and expected inflation are higher for those with more negative subjective models, with selection correction

	(1) Perceived inflation	(2) Expected inflation
end up stronger	-0.724***	-0.607***
	(0.0319)	(0.0301)
make little difference	-0.548***	-0.478***
	(0.0213)	(0.0201)
dont know	-0.452***	-0.407***
	(0.0292)	(0.0280)
Inverse Mills ratio	0.714***	0.168
	(0.248)	(0.191)
Selection stage		
r affects π in 1-2 months	0.133***	0.192***
	(0.0209)	(0.0209)
r affects π in 1-2 yrs	0.278***	0.310***
	(0.0205)	(0.0205)
Pseudo-R^2 (selection)	0.0477	0.0536
Controls	All	All
Time FE	Yes	Yes
Observations	95339	95339

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Note: The table reports the coefficients from estimating a linear regression of perceived and expected inflation on the responses to Question 1, augmented with the inverse Mills ratio from a first-stage probit regression of whether the dependent variable is observed on measures of economic literacy. The omitted category is the belief that inflation makes the economy weaker. The model is run using the 2-step limited information method in Heckman (1979). Time fixed effects and controls as in footnote 17 are included in both stages.

D Dynamic model: derivations and proofs

D.1 Proof of Lemma 1

The proof is an adaptation of the derivation of expression (34) in Maćkowiak and Wiederholt (2015). First, substitute the budget constraint (18) into the utility function (17) to obtain:

$$\tilde{\mathbb{E}}_0^i U_0^i = \tilde{\mathbb{E}}_0^i \beta^t \frac{1}{1-\frac{1}{\sigma}} \left(\frac{R_{t-1}}{\Pi_t} B_{t-1}^i + Y_t - B_t^i \right)^{1-\frac{1}{\sigma}} \tag{79}$$

Write this in log-deviations from steady state, where $\bar{X}$ denotes the steady state value of the corresponding variable X_t, and $x_t \equiv \log(X_t/\bar{X})$ is the corresponding log-deviation:

$$\tilde{\mathbb{E}}_0^i U_0^i = \tilde{\mathbb{E}}_0^i \beta^t \frac{1}{1-\frac{1}{\sigma}} \left(\bar{R}\bar{B}^i \exp(r_{t-1} - \pi_t + b_{t-1}^i) + \bar{Y}\exp(y_t) - \bar{B}^i \exp(b_t^i) \right)^{1-\frac{1}{\sigma}} \tag{80}$$

We then take a quadratic approximation of this with respect to each variable in log-deviation, about the steady state. For this, define $z_t = (r_{t-1}, \pi_t, y_t)'$ as the vector of exogenous variables taken as given by the household in period t. The past asset choice b_{t-1}^i is also taken as given in period t, and b_t^i is the only choice variable. After the quadratic approximation, expected discounted utility is given by:

$$\begin{aligned} \tilde{\mathbb{E}}_0^i U_0^i \approx \tilde{\mathbb{E}}_0^i \hat{U}_0^i = \bar{U}^i + \tilde{\mathbb{E}}_0^i \sum_{t=0}^{\infty} \beta^t \Bigg[& h_b b_t^i + h_z z_t + \frac{1}{2} H_{bb,-1} b_t^i b_{t-1}^i + \frac{1}{2} H_{bb,0} (b_t^i)^2 + \frac{1}{2} H_{bb,1} b_{t+1}^i b_t^i \\ & + \frac{1}{2} b_t^i H_{bz,0} z_t + \frac{1}{2} b_t^i H_{bz,1} z_{t+1} + \frac{1}{2} z_t' H_{zz,0} z_t + \frac{1}{2} z_t' H_{zb,-1} b_{t-1}^i + \frac{1}{2} z_t' H_{zb,0} b_t^i \Bigg] \\ & + \beta^{-1} \left(h_{-1} b_{-1}^i + \frac{1}{2} H_{-1} (b_{-1}^i)^2 + \frac{1}{2} H_{bb,1} b_{-1}^i b_0^i + \frac{1}{2} b_{-1}^i H_{bz,1} z_0 \right) \end{aligned} \tag{81}$$

where $\beta^t h_b$ denotes the first derivative of U_0^i with respect to b_t^i, evaluated at the steady state. Similarly, h_z denotes the vector of first derivatives of U_0^i with respect to z_t, evaluated at steady state. The matrices $\beta^t H_{jk,\tau}$ denote the second derivatives of U_0^i with respect to j_t and $k_{t+\tau}$, for $j_t, k_t \in \{b_t^i, z_t\}$, evaluated at steady state. $\beta^{-1} h_{-1}$ and $\beta^{-1} H_{-1}$ are the first and second derivatives of U_0^i with respect to initial wealth b_{-1}^i, evaluated at steady state. As in Maćkowiak and Wiederholt (2015), note that there are no cross-products of b_t and z_{t-1}, because from equation 80 the first derivative of U_0^i with respect to b_t^i does not depend on any elements of z_{t-1}. Similarly, there are no terms in the interaction of z_t and z_{t-1} or z_{t+1}.

We now simplify this, using several properties of the coefficient vectors and matrices. First, we have that $z_t' H_{zb,0} b_t^i = b_t^i H_{bz,0} z_t$. Second:

$$\begin{aligned} \tilde{\mathbb{E}}_0^i \sum_{t=0}^{\infty} \beta^t \frac{1}{2} H_{bb,-1} b_t^i b_{t-1}^i &= \tilde{\mathbb{E}}_0^i \sum_{t=0}^{\infty} \beta^t \frac{1}{2} \beta^{-1} H_{bb,1} b_t^i b_{t-1}^i \\ &= \frac{1}{2} \beta^{-1} H_{bb,1} b_{-1}^i b_0^i + \tilde{\mathbb{E}}_0^i \sum_{t=0}^{\infty} \beta^t \frac{1}{2} \beta^{-1} H_{bb,1} b_t^i b_{t+1}^i \end{aligned} \tag{82}$$

Similarly:

$$\begin{aligned}\tilde{\mathbb{E}}_0^i \sum_{t=0}^{\infty} \beta^t \frac{1}{2} z_t' H_{zb,-1} b_{t-1}^i &= \tilde{\mathbb{E}}_0^i \sum_{t=0}^{\infty} \beta^t \frac{1}{2} \beta^{-1} b_{t-1}^i H_{bz,1} z_t \\ &= \frac{1}{2} \beta^{-1} b_{-1}^i H_{zb,1} z_0 + \tilde{\mathbb{E}}_0^i \sum_{t=0}^{\infty} \beta^t \frac{1}{2} b_t^i H_{zb,1} z_{t+1}\end{aligned} \tag{83}$$

Using these, and the fact that $h_b = 0$, the log-quadratic approximation to utility becomes:

$$\begin{aligned}\tilde{\mathbb{E}}_0^i \hat{U}_0^i = \bar{U}^i + \tilde{\mathbb{E}}_0^i \sum_{t=0}^{\infty} \beta^t \Bigg[& h_z z_t + \frac{1}{2} H_{bb,0} (b_t^i)^2 + H_{bb,1} b_{t+1}^i b_t^i \\ & + b_t^i H_{bz,0} z_t + b_t^i H_{bz,1} z_{t+1} + \frac{1}{2} z_t' H_{zz,0} z_t \Bigg] \\ & + \beta^{-1} \Bigg(h_{-1} b_{-1}^i + \frac{1}{2} H_{-1} (b_{-1}^i)^2 + H_{bb,1} b_{-1}^i b_0^i + b_{-1}^i H_{bz,1} z_0 \Bigg)\end{aligned} \tag{84}$$

Next, we find b_t^{i*}, the optimal asset holdings chosen each period by a fully-informed household. This satisfies the first order condition:

$$\tilde{\mathbb{E}}_0^{i*} \Bigg[H_{bb,0} b_t^{i*} + H_{bb,1} b_{t+1}^{i*} + \beta^{-1} H_{bb,1} b_{t-1}^{i*} \Bigg] = -\tilde{\mathbb{E}}_0^{i*} \Bigg[H_{bz,0} z_t + H_{bz,1} z_{t+1} \Bigg] \tag{85}$$

Define the expected utility of a fully-informed household, $\tilde{\mathbb{E}}_0^{i*} \hat{U}_0^{i*}$, as the expected discounted utility if the household chooses this optimal saving behavior. The expected utility loss from deviating from this rule is:

$$\begin{aligned}\tilde{\mathbb{E}}_0^i (\hat{U}_0^{i*} - \hat{U}_0^i) = \tilde{\mathbb{E}}_0^i \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} H_{bb,0} (b_t^{i*})^2 + H_{bb,1} b_{t+1}^{i*} b_t^{i*} - \frac{1}{2} H_{bb,0} (b_t^i)^2 - H_{bb,1} b_{t+1}^i b_t^i \right] \\ + \tilde{\mathbb{E}}_0^i \sum_{t=0}^{\infty} \beta^t (b_t^{i*} - b_t^i)(H_{bz,0} z_t + H_{bz,1} z_{t+1}) + \tilde{\mathbb{E}}_0^i \beta^{-1} \Bigg(H_{bb,1} b_{-1}^i b_0^{i*} - H_{bb,1} b_{-1}^i b_0^i \Bigg)\end{aligned} \tag{86}$$

where I have used that $b_{-1}^{i*} = b_{-1}^i$.

Substituting in equation 85 we have:

$$\tilde{\mathbb{E}}_0^i(\hat{U}_0^{i*}-\hat{U}_0^i)=\tilde{\mathbb{E}}_0^i\sum_{t=0}^{\infty}\beta^t\left[\frac{1}{2}H_{bb,0}(b_t^{i*})^2+H_{bb,1}b_{t+1}^{i*}b_t^{i*}-\frac{1}{2}H_{bb,0}(b_t^i)^2-H_{bb,1}b_{t+1}^ib_t^i\right]$$
$$-\tilde{\mathbb{E}}_0^i\sum_{t=0}^{\infty}\beta^t(b_t^{i*}-b_t^i)(H_{bb,0}b_t^{i*}+H_{bb,1}b_{t+1}^{i*}+\beta^{-1}H_{bb,1}b_{t-1}^{i*})+\tilde{\mathbb{E}}_0^i\beta^{-1}\bigg(H_{bb,1}b_{-1}^ib_0^{i*}-H_{bb,1}b_{-1}^ib_0^i\bigg) \tag{87}$$

Collecting terms and rearranging:

$$\begin{aligned}\tilde{\mathbb{E}}_0^i(\hat{U}_0^{i*}-\hat{U}_0^i)=\tilde{\mathbb{E}}_0^i\sum_{t=0}^{\infty}\beta^t\bigg[&-\frac{1}{2}H_{bb,0}(b_t^{i*})^2-\frac{1}{2}H_{bb,0}(b_t^i)^2+H_{bb,0}b_t^ib_t^{i*}+H_{bb,1}b_{t+1}^{i*}b_t^i\\&-H_{bb,1}b_{t+1}^ib_t^i\bigg]+\tilde{\mathbb{E}}_0^i\sum_{t=0}^{\infty}\beta^t\beta^{-1}H_{bb,1}b_{t-1}^{i*}(b_t^i-b_t^{i*})+\tilde{\mathbb{E}}_0^i\beta^{-1}H_{bb,1}b_{-1}^i(b_0^{i*}-b_0^i)\end{aligned} \tag{88}$$

The second summation can be written as:

$$\begin{aligned}\tilde{\mathbb{E}}_0^i\sum_{t=0}^{\infty}\beta^t\beta^{-1}H_{bb,1}b_{t-1}^{i*}(b_t^i-b_t^{i*})=\beta^{-1}H_{bb,1}b_{-1}^ib_0^i-\tilde{\mathbb{E}}_0^i\beta^{-1}H_{bb,1}b_{-1}^ib_0^{i*}&\\+\tilde{\mathbb{E}}_0^i\sum_{t=0}^{\infty}\beta^tH_{bb,1}b_t^{i*}(b_{t+1}^i-b_{t+1}^{i*})&\end{aligned} \tag{89}$$

where I have again used $b_{-1}^{i*}=b_{-1}^i$.

Substituting this into the expected utility loss and collecting terms:

$$\tilde{\mathbb{E}}_0^i(\hat{U}_0^{i*}-\hat{U}_0^i)=\tilde{\mathbb{E}}_0^i\sum_{t=0}^{\infty}\beta^t\left[-\frac{1}{2}H_{bb,0}(b_t^i-b_t^{i*})^2-H_{bb,1}(b_t^i-b_t^{i*})(b_{t+1}^i-b_{t+1}^{i*})\right] \tag{90}$$

Differentiating the instantaneous utility function $U_{p,t}$ twice gives:

$$H_{bb,0}=\frac{\partial^2U_{p,t}^i}{\partial(b_t^i)^2}\bigg|_{ss}=-\frac{1}{\sigma}(\bar{C}^i)^{-\frac{1}{\sigma}-1}(\bar{B}^i)^2(1+\beta^{-1}),\quad H_{bb,1}=\frac{\partial^2U_{p,t}^i}{\partial b_t^i\partial b_{t+1}^i}\bigg|_{ss}=\frac{1}{\sigma}(\bar{C}^i)^{-\frac{1}{\sigma}-1}(\bar{B}^i)^2 \tag{91}$$

Therefore:

$$\tilde{\mathbb{E}}_0^i(\hat{U}_0^{i*}-\hat{U}_0^i)=-\frac{1}{\sigma}(\bar{C}^i)^{-\frac{1}{\sigma}-1}(\bar{B}^i)^2\tilde{\mathbb{E}}_0^i\sum_{t=0}^{\infty}\beta^t\left[-\frac{1+\beta^{-1}}{2}(b_t^i-b_t^{i*})^2+(b_t^i-b_t^{i*})(b_{t+1}^i-b_{t+1}^{i*})\right] \tag{92}$$

Next, we transform this into an equation involving consumption choices, rather than asset choices. Log-linearizing the budget constraint (18) gives:

$$\bar{C}^i c_t^i = \beta^{-1}\bar{B}^i(r_{t-1} - \pi_t + b_{t-1}^i) - \bar{B}^i b_t^i + \bar{Y} y_t \tag{93}$$

Subtracting the equivalent for the fully-informed household:

$$\bar{C}^i(c_t^i - c_t^{i*}) = \beta^{-1}\bar{B}^i(b_{t-1}^i - b_{t-1}^{i*}) - \bar{B}^i(b_t^i - b_t^{i*}) \tag{94}$$

We substitute this into equation 92 and rearrange. To see how the rearrangement works, define $\Delta_t^i = \bar{B}^i/\bar{C}^i \cdot (b_t^i - b_t^{i*})$, so that equation 94 becomes:

$$\Delta_t^i = \beta^{-1}\Delta_{t-1}^i - (c_t^i - c_t^{i*}) \tag{95}$$

Substituting out for $(b_t^i - b_t^{i*})$ and $(b_{t+1}^i - b_{t+1}^{i*})$ in equation 92 using the definition of Δ_t^i gives:

$$\tilde{\mathbb{E}}_0^i(\hat{U}_0^{i*} - \hat{U}_0^i) = -\frac{1}{\sigma}(\bar{C}^i)^{1-\frac{1}{\sigma}}\tilde{\mathbb{E}}_0^i\sum_{t=0}^{\infty}\beta^t\left[-\frac{1+\beta^{-1}}{2}(\Delta_t^i)^2 + \Delta_t^i\Delta_{t+1}^i\right] \tag{96}$$

The terms inside the square brackets can be rearranged to:

$$\begin{aligned}
-\frac{1}{2}(\Delta_t^i)^2 - \frac{1}{2\beta}(\Delta_t^i)^2 + \Delta_t^i\Delta_{t+1}^i &= -\frac{1}{2}\frac{1}{\beta^2}(\Delta_{t-1}^i)^2 + \frac{1}{\beta}\Delta_{t-1}^i(c_t^i - c_t^{i*}) - \frac{1}{2}(c_t^i - c_t^{i*})^2 \\
&\quad - \frac{1}{2\beta}(\Delta_t^i)^2 + \Delta_t^i\Delta_{t+1}^i \\
= -\frac{1}{2}\frac{1}{\beta^2}(\Delta_{t-1}^i)^2 + \frac{1}{\beta}\Delta_{t-1}^i(c_t^i - c_t^{i*}) &- \frac{1}{2}(c_t^i - c_t^{i*})^2 - \frac{1}{2\beta}(\Delta_t^i)^2 + \Delta_t^i(\beta^{-1}\Delta_t^i - (c_{t+1}^i - c_{t+1}^{i*})) \\
&= -\frac{1}{2}(c_t^i - c_t^{i*})^2 + \frac{1}{2\beta}\big((\Delta_t^i)^2 - \frac{1}{\beta}(\Delta_{t-1}^i)^2\big) - \big(\Delta_t^i(c_{t+1}^i - c_{t+1}^{i*}) - \frac{1}{\beta}\Delta_{t-1}^i(c_t^i - c_t^{i*})\big)
\end{aligned} \tag{97}$$

where the first and second equalities involve substituting out using equation 95.

Substituting this into equation 96, canceling terms when they appear from multiple periods, and noting that $\Delta_{-1}^i = 0$, we obtain:

$$\tilde{\mathbb{E}}_0^i(\hat{U}_0^{i*} - \hat{U}_0^i) = \frac{1}{2\sigma}(\bar{C}^i)^{1-\frac{1}{\sigma}}\tilde{\mathbb{E}}_0^i\sum_{t=0}^{\infty}\beta^t(c_t^i - c_t^{i*})^2 \tag{98}$$

D.2 Supply-side derivations

There is a perfectly competitive final goods producer which combines intermediate goods varieties with a CES production function. The demand facing an intermediate goods producer j with price P_t^j is therefore:

$$Y_t^j = \left(\frac{P_t^j}{P_t}\right)^{-\varepsilon} Y_t \tag{99}$$

where ε is the elasticity of substitution between intermediate varieties, Y_t is aggregate production and:

$$P_t = \left(\int (P_t^j)^{1-\varepsilon} dj\right)^{\frac{1}{1-\varepsilon}} \tag{100}$$

is the aggregate price level. Intermediate goods firms are therefore monopolistic. They produce using labor as their only input, according to a linear production function:

$$Y_t^j = A_t N_t^j \tag{101}$$

where A_t is exogenous total factor productivity, common to all firms j.

The profit maximization problem of intermediate goods producer j in period t is therefore:

$$\max_{P_t^j, N_t^j} \mathbb{E}_t \sum_{s=0}^{\infty} \Lambda_{t,t+s} \left[P_{t+s}^j Y_{t+s}^j - W_{t+s} N_{t+s}^j - \frac{\Psi}{2}\left(\frac{P_t^j}{P_{t-1}^j} - 1\right)^2 P_t Y_t \right] \tag{102}$$

subject to the demand function (99) and the production function (101). Ψ is a positive constant, reflecting the degree of price stickiness. Firms have rational expectations. They are owned by a mutual fund which pays out equally to all households, so the stochastic discount factor is:

$$\Lambda_{t,t+s} = \beta^s \left(\frac{\bar{C}_{t+s}}{\bar{C}_t}\right)^{-\frac{1}{\sigma}} \tag{103}$$

It is well-known that solving this firm problem, noting that all firms are identical so choose the same price, and then log-linearizing about the steady state with $\Pi_t = P_t/P_{t-1} = 1$, gives:[35]

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \frac{\varepsilon - 1}{\Psi} mc_t \tag{104}$$

where $mc_t = W_t/(A_t P_t)$ is real marginal costs. This corresponds to equation 22, with $\kappa = (\varepsilon - 1)/\Psi$.

[35] See e.g. Rotemberg (1987), Ascari and Rossi (2012).

The labor packer aggregates varieties from labor unions with a CES function, so the demand facing a labor union k with nominal wage W_t is:

$$N_t^k = \left(\frac{W_t^k}{W_t}\right)^{-\varepsilon_N} N_t \tag{105}$$

where ε_N is the elasticity of substitution between labor varieties, N_t is aggregate labor supply, and:

$$W_t = \left(\int (W_t^k)^{1-\varepsilon_N} dk\right)^{\frac{1}{1-\varepsilon_N}} \tag{106}$$

is the aggregate nominal wage.

Labor unions maximize wage revenue, net of an increasing convex disutility from supplying labor of $\nu(N_t^k)$. They are also subject to quadratic costs of adjusting real wages, and are owned by all households, giving the same stochastic discount factor $\Lambda_{t,t+s}$ as firms. The problem of a labor union k in period t is therefore:

$$\max_{W_t^k, N_t^k} \mathbb{E}_t \sum_{s=0}^{\infty} \Lambda_{t,t+s} \left[W_{t+s}^k N_{t+s}^k - \nu(N_{t+s}^k) - \frac{\Psi_N}{2} \left(\frac{W_t^k}{W_{t-1}^k} \frac{P_{t-1}}{P_t} - 1 \right)^2 W_t N_t \right] \tag{107}$$

I take the real wage rigidity parameter $\Psi_N \to \infty$, so the the solution to the union problem is to set constant real wages:

$$\frac{W_t^k}{P_t} = \frac{W_{t-1}^k}{P_{t-1}} \tag{108}$$

All unions are therefore identical, and the log-linearized solution for real marginal costs is:

$$mc_t = w_t - p_t - a_t = -a_t \tag{109}$$

Finally, I derive equation 25 with a guess-and-verify approach. Guess that the law of motion for inflation is as in equation 25, where $\mathbb{E}_t v_{\pi t+1} = 0$. The rational expectation of π_{t+1} is therefore $\mathbb{E}_t \pi_{t+1} = \rho_\pi \pi_t$. Substituting this and equation 109 into equation 22 gives:

$$\pi_t = -\frac{\kappa}{1 - \beta \rho_\pi} a_t \tag{110}$$

Assume TFP follows the exogenous AR(1) process:

$$a_t = \rho_a a_{t-1} + v_{at} \tag{111}$$

This then implies that:

$$\pi_t = \rho_a \pi_{t-1} - \frac{\kappa}{1 - \beta\rho_\pi} v_{at} \tag{112}$$

which verifies the guess and implies $\rho_\pi = \rho_a$.

D.3 Forecasts using the subjective model

The subjective model represented by equations 26 - 28 can be written in VAR form as:

$$Y_t = A^i Y_{t-1} + B^i U_t \tag{113}$$

where:

$$\begin{aligned} Y_t = (\pi_t, y_t, i_t)', \quad A &= \begin{bmatrix} \rho_\pi^i & 0 & 0 \\ (\alpha^i + \lambda^i \phi^i)\rho_\pi^i & \rho_y^i & 0 \\ \phi\rho_\pi & 0 & 0 \end{bmatrix} \\ U_t = (u_{\pi t}, u_{yt}, u_{it})', \quad B &= \begin{bmatrix} 1 & 0 & 0 \\ \alpha^i + \lambda^i \phi^i & 1 & \lambda^i \\ \phi^i & 0 & 1 \end{bmatrix} \end{aligned} \tag{114}$$

To form a forecast of future variables, the fully-informed agent uses:

$$\tilde{\mathbb{E}}_t^{i*} Y_{t+s} = (A^i)^s Y_t \tag{115}$$

That is, their forecasts are optimal given their subjective model. To find $(A^i)^s$, first find diagonal matrix D^i and matrix P^i such that:

$$A^i = P^i D^i (P^i)^{-1} \tag{116}$$

This is satisfied with:

$$P = \begin{bmatrix} 0 & (\phi^i)^{-1} & 0 \\ 0 & \dfrac{(\alpha^i + \lambda^i \phi^i)\rho_\pi^i}{\phi^i(\rho_\pi^i - \rho_y^i)} & 1 \\ 1 & 1 & 0 \end{bmatrix}, \quad D^i = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \rho_\pi^i & 0 \\ 0 & 0 & \rho_y^i \end{bmatrix} \tag{117}$$

We then have that:

$$
\begin{aligned}
(A^i)^s = P^i(D^i)^s(P^i)^{-1} &= P^i \cdot \begin{bmatrix} 0^s & 0 & 0 \\ 0 & (\rho_\pi^i)^s & 0 \\ 0 & 0 & (\rho_y^i)^s \end{bmatrix} \cdot (P^i)^{-1} \\
&= \begin{bmatrix} (\rho_\pi^i)^s & 0 & 0 \\ \dfrac{(\alpha^i + \lambda^i\phi^i)\rho_\pi^i}{\rho_\pi^i - \rho_y^i}((\rho_\pi^i)^s - (\rho_y^i)^s) & (\rho_y^i)^s & 0 \\ \phi^i(\rho_\pi^i)^s & 0 & 0 \end{bmatrix}
\end{aligned} \tag{118}
$$

This implies equations 32 - 34.

D.4 Proposition 4

Aggregating over household consumption functions (35), and using equation 24, we have:

$$
y_t = \bar{\Theta} y_t - \sigma\beta r_t + \int \omega^i \frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t} \tilde{\mathbb{E}}_t^i \pi_t di \tag{119}
$$

where ω^i is a household weight as in equation 11, and:

$$
\bar{\Theta} = \int \omega^i \frac{1-\beta}{1-\beta\rho_y^i} di \tag{120}
$$

Rearranging, and substituting out for $\tilde{\mathbb{E}}_t^i \pi_t$ using equation 30, yields equation 36, with:

$$
\begin{aligned}
\alpha = \frac{1}{1-\bar{\Theta}} \int \omega^i \frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t} K^i di, \quad \lambda = -\frac{\sigma\beta}{1-\bar{\Theta}}, \\
\rho_y = \frac{1}{1-\bar{\Theta}}, \quad \hat{\omega}^i = \omega^i \rho_\pi^i \frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t}(1-K^i)
\end{aligned} \tag{121}
$$

This proves Proposition 4.

Using this law of motion, and equation 23, y_{t-1} can be written as:

$$
y_{t-1} = \left[\lambda\phi + \rho_y \int \omega^i \frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t} K^i di\right] \pi_{t-1} + \rho_y \int \hat{\omega}^i \tilde{\mathbb{E}}_{t-2}^i \pi_{t-2} di \tag{122}
$$

Using equation 30, the lagged term in the true law of motion can be written as:

$$\int \hat{\omega}^i \tilde{\mathbb{E}}^i_{t-1}\pi_{t-1} di = \left[\int \hat{\omega}^i K^i di\right]\pi_{t-1} + \int \hat{\omega}^i \rho^i_\pi (1-K^i)\tilde{\mathbb{E}}^i_{t-2}\pi_{t-2} di \tag{123}$$

The lagged terms in the subjective model (122) and the true law of motion (123) for y_t are both therefore determined by realized π_{t-1} and a weighted average of $\tilde{\mathbb{E}}^i_{t-2}\pi_{t-2}$, with the weights increasing in $\hat{\omega}^i$. While not exactly the same, equation 28 therefore provides a tractable approximation to the functional form of 36.

D.5 Microfounding the imposed properties of information and subjective models

First, I show that the subjective model properties defined in Section 4.6 are sufficient for the model to qualitatively match empirical Results 2 and 3. I begin with Result 3. Subjective model property 2 implies that:

$$\frac{\partial}{\partial \tilde{\mathbb{E}}^i_t \pi_t}\left(\frac{\partial c^i_t}{\partial \tilde{\mathbb{E}}^i_t \pi_t}\right) = \frac{\partial \hat{\alpha}^i_t}{\partial \tilde{\mathbb{E}}^i_t \pi_t} \cdot \frac{\partial}{\partial \hat{\alpha}^i_t}\left(\frac{\partial c^i_t}{\partial \tilde{\mathbb{E}}^i_t \pi_t}\right) = \frac{\partial \hat{\alpha}^i_t}{\partial \tilde{\mathbb{E}}^i_t \pi_t} \cdot \frac{\beta(1-\beta)\rho^i_\pi}{(1-\beta\rho^i_\pi)(1-\beta\rho^i_y)} < 0 \tag{124}$$

where the second equality makes use of equation 35. Higher perceived inflation is therefore associated with more negative beliefs about the impact of inflation on consumption.

For Result 2, note that from equation 30:

$$\frac{\partial}{\partial \pi_t}\left(\frac{\partial c^i_t}{\partial \tilde{\mathbb{E}}^i_t \pi_t}\right) = K^i \frac{\partial}{\partial \tilde{\mathbb{E}}^i_t \pi_t}\left(\frac{\partial c^i_t}{\partial \tilde{\mathbb{E}}^i_t \pi_t}\right) \tag{125}$$

Combining this with equation 124, and the fact that $K^i \geq 0$, we have that:

$$\frac{\partial}{\partial \pi_t}\left[\Pr\left(\left.\frac{\partial c^i_t}{\partial \tilde{\mathbb{E}}^i_t \pi_t}\right|_{\hat{\alpha}^i_t} < X\right)\right] \geq 0 \tag{126}$$

for any threshold X. The inequality is strict if $K^i > 0$, and is an equality otherwise. As realized inflation rises, each household becomes more likely to report that inflation makes the economy weaker.

Next, I show how a model with information costs leads to equation 37. Substituting the consumption functions of informed and uninformed households (equation 35) into the expected utility loss from imperfect information (equation 20) gives:

$$\tilde{\mathbb{E}}_0^i(\hat{U}_0^{i*} - \hat{U}_0^i) = \frac{(\bar{C}^i)^{1-\frac{1}{\sigma}}}{2\sigma}\left(\frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t}\right)^2 \tilde{\mathbb{E}}_0^i \sum_{t=0}^{\infty} \beta^t (\pi_t - \tilde{\mathbb{E}}_t^i \pi_t)^2 \tag{127}$$

where:

$$\frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t} = \frac{\beta \rho_\pi^i [(1-\beta)(\alpha^i + \lambda^i \phi^i) - \sigma(\beta\phi^i - 1)(1 - \beta\rho_y^i)]}{(1-\beta\rho_\pi^i)(1-\beta\rho_y^i)} \tag{128}$$

is the elasticity of the household's consumption to perceived inflation, under the initial subjective model held at the start of the period.

Following the rational inattention literature I assume that increasing information precision is costly to the household. The utility cost of a signal s_t^i is given by:

$$\mathcal{C}(\{s_t^i\}^t) = \psi \sum_{t=0}^{\infty} \beta^t I(\pi^t; s_t^i | \mathcal{I}_{t-1}^i) \tag{129}$$

where $\psi > 0$ is a positive constant and $I(\pi^t; s_t^i|\mathcal{I}_{t-1}^i)$ is the Shannon mutual information between priors and posteriors in period t. This cost function is common in the rational inattention literature (Maćkowiak et al., 2020).

To solve for optimal information processing, I make the simplifying assumption that the household chooses information as if they are certain about the parameters of their subjective model. Similarly, they ignore that they will update those parameters after receiving information. This is akin to the anticipated utility assumption in many models with least-squares learning, where agents do not consider that their perceived law of motion will change as they observe new periods of data in the future (see Bullard and Suda (2016) for a discussion).

The household information choice problem then has the same form as the firm's rational inattention problem in Maćkowiak and Wiederholt (2009). As is standard in the rational inattention literature, I make the following further assumptions:

Assumption 1: (π_t, s_t^i) has a stationary Gaussian distribution.

Assumption 2: When the household decides on their information strategy in period 0, they receive a long sequence of signals of their chosen form. This implies that $\tilde{\mathbb{E}}_t^i(\pi_t^2|\mathcal{I}_t^i)$ is constant over time.

Assumption 3: In period t, households can only process information about variables realized up to period t. They cannot process any information about realizations of inflation in future periods.[36]

[36]This ensures that as the cost of information approaches zero the household information set in period

Under these assumptions, Maćkowiak and Wiederholt (2009) show that the optimal signal is of the form in equation 29. Using that signal structure, the utility cost of period-t signal s_t^i is:

$$\begin{aligned} I(\pi^t; s_t^i | \mathcal{I}_{t-1}^i) \equiv H(\pi_t | s^{t-1,i}) - H(\pi_t | s^{t,i}) &= \frac{1}{2} \log_2 \left(\frac{Var(\pi_t | \mathcal{I}_{t-1}^i)}{Var(\pi_t | \mathcal{I}_t^i)} \right) \\ &= \frac{1}{2} \log_2 \left(\frac{1}{1 - K^i} \right) \end{aligned} \tag{130}$$

where the final equality uses standard properties of the Kalman filter.

Assumption 2 implies that $\tilde{\mathbb{E}}_0^i (\pi_t - \mathbb{E}_t \pi_t)^2$ is constant over time. From the properties of the Kalman filter:

$$\tilde{\mathbb{E}}_0^i (\pi_t - \mathbb{E}_t \pi_t)^2 = \frac{(1 - K^i)\sigma_\pi^2}{1 - (\rho_\pi^i)^2 (1 - K^i)} \tag{131}$$

Using these results, and evaluating the resulting geometric series in the utility losses and costs of information, the household information choice problem reduces to:

$$\min_K \frac{(\bar{C}^i)^{1-\frac{1}{\sigma}}}{2\sigma} \left(\frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t} \right)^2 \frac{(1 - K^i)\sigma_\pi^2}{1 - (\rho_\pi^i)^2 (1 - K^i)} + \frac{\psi}{2} \log_2 \left(\frac{1}{1 - K^i} \right) \tag{132}$$

subject to $K^i \in [0, 1]$. The first order condition for an interior solution is:

$$\frac{1 - K^i}{(1 - (\rho_\pi^i)^2 (1 - K^i))^2} = \frac{\delta^{i*}}{(1 - (\rho_\pi^i)^2)^2} \left(\frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t} \right)^{-2} \tag{133}$$

where:

$$\delta^{i*} = \frac{2\psi\sigma(1 - (\rho_\pi^i)^2)^2}{(\bar{C}^i)^{1-\frac{1}{\sigma}} \sigma_\pi^2 \ln(2)} \tag{134}$$

Since $\delta^{i*} > 0$, the K^i implied by this first order condition is always strictly less than 1. Finally, we find the region where $K^i \geq 0$ binds. Differentiating the left hand side of equation 133 with respect to K^i gives:

$$\frac{\partial}{\partial K^i} \left(\frac{1 - K^i}{(1 - (\rho_\pi^i)^2 (1 - K^i))^2} \right) = -\frac{1 + (\rho_\pi^i)^2 (1 - K^i)}{(1 - (\rho_\pi^i)^2 (1 - K^i))^3} < 0 \tag{135}$$

The left hand side of equation 133 is therefore strictly decreasing in K^i. The constraint

t contains realized values of all period t variables, but not realizations of variables in future periods, as in standard full-information models. See Jurado (2021) for a detailed discussion of this assumption.

therefore binds whenever the right hand side is sufficiently large, that is when:

$$\left(\frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t}\right)^2 < \delta^{i*} \tag{136}$$

This completes the derivation of equation 37.

Finally, I provide a microfoundation for equation 38. The household faces Knightian uncertainty about the α^i parameter in their subjective model. After observing the realization of s_t^i, the household updates their subjective model to reflect this: following the literature on ambiguity aversion they make decisions using worst-case beliefs (Hansen and Sargent, 2008).

Formally, the household selects beliefs and actions as if they are playing a game with an 'evil agent', who distorts α^i to minimize expected utility, while the household simultaneously chooses c_t^i to maximize expected utility. The maximization problem is solved by the consumption function in equation 35 with the updated $\hat{\alpha}_t^i$.

To solve the evil agent problem, we then need to find the indirect expected utility when households follow this consumption function. Begin with the expected utility of a household who is fully-informed about inflation each period. To simplify the problem, here I assume that $\sigma \to 1$, so the instantaneous utility from consumption C_t^i is $\log(C_t^i)$. The log-quadratic approximation of expected discounted utility, substituting in the consumption function of the informed household, is therefore:

$$\begin{aligned}\tilde{\mathbb{E}}_0^{i*}\hat{U}_0^{i*} = \tilde{\mathbb{E}}_0^{i*}\sum_{t=0}^{\infty}\beta^t\Bigg(&\frac{1-\beta}{1-\beta\rho_y^i}y_t - \sigma\beta r_t \\ &+ \frac{\beta\rho_\pi^i[(1-\beta)(\alpha^i+\lambda^i\phi^i) - \sigma(\phi^i\beta-1)(1-\beta\rho_y^i)]}{(1-\beta\rho_\pi^i)(1-\beta\rho_y^i)}\pi_t\Bigg)\end{aligned} \tag{137}$$

Substituting out for expected future inflation, interest rates, and real income using the subjective model (equations 32 - 34) gives indirect utility as a function of current observables and subjective model parameters:

$$\tilde{\mathbb{E}}_0^{i*}\hat{U}_0^{i*} = \frac{1-\beta}{(1-\beta\rho_y^i)^2}y_0 - \sigma\beta r_0 + \frac{1}{1-\beta\rho_\pi^i}\left(\frac{\beta\rho_\pi^i(\alpha^i+\lambda^i\phi^i)}{1-\beta\rho_y^i} - \sigma\beta^2\phi^i\rho_\pi^i + \frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i\pi_t}\right)\pi_0 \tag{138}$$

Finally, use the expression for the expected utility loss from limited information (equation

127) to find the expected indirect utility of the potentially uninformed household:

$$\tilde{\mathbb{E}}_0^i \hat{U}_0^i = \frac{1-\beta}{(1-\beta\rho_y^i)^2} y_0 - \sigma\beta r_0 + \frac{1}{1-\beta\rho_\pi^i}\left(\frac{\beta\rho_\pi^i(\alpha^i+\lambda^i\phi^i)}{1-\beta\rho_y^i} - \sigma\beta^2\phi^i\rho_\pi^i + \frac{\partial c_t^i}{\partial\tilde{\mathbb{E}}_t^i\pi_t}\right)\tilde{\mathbb{E}}_0^i\pi_0 \\ - \frac{\log(\bar{C}^i)}{2(1-\beta)}\left(\frac{\partial c_t^i}{\partial\tilde{\mathbb{E}}_t^i\pi_t}\right)^2 \frac{(1-K^i)\sigma_\pi^2}{1-(\rho_\pi^i)^2(1-K^i)} \tag{139}$$

where I have used that the expected variance of inflation perception gaps is constant (equation 131). Differentiating the expected indirect utility with respect to α^i gives:

$$\frac{\partial\tilde{\mathbb{E}}_0^i\hat{U}_0^i}{\partial\alpha^i} = \frac{\beta(2-\beta)\rho_\pi^i}{(1-\beta\rho_\pi^i)(1-\beta\rho_y^i)}\tilde{\mathbb{E}}_0^i\pi_0 - \frac{\beta\rho_\pi^i\log(\bar{C}^i)(1-K^i)\sigma_\pi^2}{(1-\beta\rho_\pi^i)(1-\beta\rho_y^i)(1-(\rho_\pi^i)^2(1-K^i))}\cdot\frac{\partial c_t^i}{\partial\tilde{\mathbb{E}}_t^i\pi_t} \tag{140}$$

Expected indirect utility is therefore increasing in α^i if and only if perceived inflation is sufficiently high, and otherwise it is decreasing.[37] Intuitively, when perceived inflation is high, the worst case is that high inflation is associated with low real incomes. However, when perceived inflation is lower, the reverse is true. The worst case is then that inflation supports real incomes, and so the ambiguity averse household distorts their subjective model in that direction, with a positive $\hat{\alpha}_t^i$.

Formally, suppose that the evil agent chooses the distorted $\hat{\alpha}_t^i$ to minimize expected utility net of a quadratic cost of distortions from α_0^i:

$$\hat{\alpha}_0^i = \arg\min_\alpha \tilde{\mathbb{E}}_0^i\hat{U}_0^i + \frac{\psi_\alpha}{2}(\alpha-\alpha_0^i)^2 \tag{141}$$

The first order condition is:

$$\hat{\alpha}_0^i = \alpha_0^i + \alpha_1^i\tilde{\mathbb{E}}_0^i\pi_0 + \alpha_2^i\frac{\partial c_t^i}{\partial\tilde{\mathbb{E}}_t^i\pi_t} \tag{142}$$

where:

$$\alpha_1^i = -\frac{\beta(2-\beta)\rho_\pi^i}{\psi_\alpha(1-\beta\rho_\pi^i)(1-\beta\rho_y^i)} < 0 \tag{143}$$

[37]Note that since K^i is decided before any distortion to α^i, it is also not a function of expected inflation. Everything on the right hand side of condition **??** is a function of underlying parameters and the parameters of the subjective model only.

$$\alpha_2^i = -\frac{\beta\rho_\pi^i \log(\bar{C}^i)(1-K^i)\sigma_\pi^2}{\psi_\alpha(1-\beta\rho_\pi^i)(1-\beta\rho_y^i)(1-(\rho_\pi^i)^2(1-K^i))} \tag{144}$$

In all calibrations used in the paper, $\bar{C}^i = 1$ for all households i, which implies $\alpha_2^i = 0$. In that case equation 142 gives equation 38.

D.6 Selection and amplification

Equation 44 gives the difference between the elasticity of aggregate consumption to inflation with heterogeneous and homogeneous information. The selection effect therefore amplifies aggregate transmission of inflation shocks if this covariance has the same sign as $d\bar{c}_t/d\pi_t$. While this will be true for most distributions of subjective models, as discussed above, it is possible to construct counter-examples where the selection effect weakens the transmission of inflation shocks, so gives a larger role to information frictions in aggregate outcomes.

For example, consider the case where the distributions of $\partial c_t^i/\partial\tilde{\mathbb{E}}_t^i\pi_t$ with positive and negative reactions to realized inflation are exact mirror images of one another. Denoting $\phi(\cdot)$ as the PDF of $\partial c_t^i/\partial\tilde{\mathbb{E}}_t^i\pi_t$:

$$\phi\left(\frac{\partial c_t^i}{\partial\tilde{\mathbb{E}}_t^i\pi_t}\right) = \phi\left(-\frac{\partial c_t^i}{\partial\tilde{\mathbb{E}}_t^i\pi_t}\right) \quad \text{for all } \frac{\partial c_t^i}{\partial\tilde{\mathbb{E}}_t^i\pi_t} \text{ such that } K^i > 0 \tag{145}$$

That is, among the households paying positive amounts of attention to inflation, the distributions of $\partial c_t^i/\partial\tilde{\mathbb{E}}_t^i\pi_t$ above and below 0 are symmetric. This means that the partial-equilibrium response of aggregate consumption (holding r_t, y_t constant) is 0:

$$\begin{aligned}\frac{\partial\bar{c}_t}{\partial\pi_t} &= \int_0^{P_0/2}\omega^i\frac{\partial c_t^i}{\partial\tilde{\mathbb{E}}_t^i\pi_t}K^i di + \int_{P_0/2}^{P_0}\omega^i\frac{\partial c_t^i}{\partial\tilde{\mathbb{E}}_t^i\pi_t}K^i di \\ &= \int_0^{P_0/2}\omega^i\frac{\partial c_t^i}{\partial\tilde{\mathbb{E}}_t^i\pi_t}K^i di - \int_0^{P_0/2}\omega^i\frac{\partial c_t^i}{\partial\tilde{\mathbb{E}}_t^i\pi_t}K^i di = 0\end{aligned} \tag{146}$$

The average Kalman gain K^i is however positive, and so the equivalent response of aggregate consumption in the homogeneous-K^i model is:

$$\begin{aligned}\left.\frac{\partial\bar{c}_t}{\partial\pi_t}\right|_{K^i=\bar{K}} &= \bar{K}\left[\int_0^{P_0}\omega^i\frac{\partial c_t^i}{\partial\tilde{\mathbb{E}}_t^i\pi_t}di + \int_{P_0}^{1}\omega^i\frac{\partial c_t^i}{\partial\tilde{\mathbb{E}}_t^i\pi_t}di\right] \\ &= \bar{K}\int_{P_0}^{1}\omega^i\frac{\partial c_t^i}{\partial\tilde{\mathbb{E}}_t^i\pi_t}di\end{aligned} \tag{147}$$

and the full-information response is:

$$\left.\frac{\partial \bar{c}_t}{\partial \pi_t}\right|_{K^i=1} = \int_{P_0}^{1} \omega^i \frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t} di \tag{148}$$

If the average $\partial c_t^i / \partial \tilde{\mathbb{E}}_t^i \pi_t$ among the households paying no attention is non-zero, then the aggregate consumption response with a homogeneous-K^i will be non-zero, and closer to the full-information benchmark. This is because the link between the sign of $\partial c_t^i / \partial \tilde{\mathbb{E}}_t^i \pi_t$ and the covariance in equation 44 breaks down at $K^i = 0$.

E Section 6 proofs and further results

E.1 Consumption function with time-varying long-run inflation

Changing the subjective model of inflation does not change anything about the model before the initial consumption function of a fully informed household (equation 21).[38]

However, the change in subjective model to include long-run inflation $\bar{\pi}_t$ does affect how we evaluate the expectation terms. Specifically, the subjective model in VAR(1) form is now:

$$Y_t = A^i Y_{t-1} + B^i U_t \tag{149}$$

where:

$$\begin{aligned} Y_t = (\pi_t, y_t, i_t, \bar{\pi}_t)' \quad A &= \begin{bmatrix} \rho_\pi^i & 0 & 0 & 1-\rho_\pi^i \\ (\alpha^i + \lambda^i \phi^i)\rho_\pi^i & \rho_y^i & 0 & (\alpha^i + \lambda^i \phi^i)(1-\rho_\pi^i) \\ \phi \rho_\pi & 0 & 0 & \phi^i (1-\rho_\pi^i) \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ U_t = (u_{\pi t}, u_{yt}, u_{it}, v_t)' \quad B &= \begin{bmatrix} 1 & 0 & 0 & 1-\rho_\pi^i \\ \alpha^i + \lambda^i \phi^i & 1 & \lambda^i & (\alpha^i + \lambda^i \phi^i)(1-\rho_\pi^i) \\ \phi^i & 0 & 1 & \phi^i(1-\rho_\pi^i) \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned} \tag{150}$$

This is the same for the case where $\bar{\pi}_t$ is a random walk and where it is assumed to be constant. In the latter case, simply set $\sigma_v^2 = 0$.

[38] Note we assume this fully-informed household observes $\bar{\pi}_t$ as well as π_t.

To form a forecast of future variables, the fully-informed agent uses:

$$\tilde{\mathbb{E}}_t^{i*} Y_{t+s} = (A^i)^s Y_t \tag{151}$$

To find $(A^i)^s$, first find diagonal matrix D^i and matrix P^i such that:

$$A^i = P^i D^i (P^i)^{-1} \tag{152}$$

This is satisfied with:

$$P = \begin{bmatrix} 1 & 0 & (\phi^i)^{-1} & 0 \\ \dfrac{\alpha^i + \lambda^i \phi^i}{1-\rho_y^i} & 0 & \dfrac{(\alpha^i + \lambda^i \phi^i)\rho_\pi^i}{\phi^i(\rho_\pi^i - \rho_y^i)} & 1 \\ \phi^i & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad D^i = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_\pi^i & 0 \\ 0 & 0 & 0 & \rho_y^i \end{bmatrix} \tag{153}$$

We then have that:

$$(A^i)^s = \begin{bmatrix} (\rho_\pi^i)^s & 0 & 0 & 1-(\rho_\pi^i)^s \\ \dfrac{(\alpha^i + \lambda^i \phi^i)\rho_\pi^i}{\rho_\pi^i - \rho_y^i}((\rho_\pi^i)^s - (\rho_y^i)^s) & (\rho_y^i)^s & 0 & \Lambda^i(s) \\ \phi^i(\rho_\pi^i)^s & 0 & 0 & \phi^i(1-(\rho_\pi^i)^s) \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{154}$$

where

$$\Lambda^i(s) = \frac{(\alpha^i + \lambda^i \phi^i)\big(\rho_\pi^i - \rho_y^i - (\rho_\pi^i)^{s+1}(1-\rho_y^i) + (\rho_y^i)^{s+1}(1-\rho_\pi^i)\big)}{(\rho_\pi^i - \rho_y^i)(1-\rho_y^i)} \tag{155}$$

Using this to evaluate the infinite sums in the consumption function (21) gives:

$$\begin{aligned} c_t^{i*} = \frac{1-\beta}{1-\beta\rho_y^i} y_t - \sigma\beta r_t &+ \frac{\beta\rho_\pi^i[(1-\beta)(\alpha^i + \lambda^i\phi^i) - \sigma(\phi^i\beta - 1)(1-\beta\rho_y^i)]}{(1-\beta\rho_\pi^i)(1-\beta\rho_y^i)}\pi_t \\ &+ \frac{\beta(1-\rho_\pi^i)[(1-\beta)(\alpha^i + \lambda^i\phi^i) - \sigma(\phi^i\beta - 1)(1-\beta\rho_y^i)]}{(1-\beta)(1-\beta\rho_\pi^i)(1-\beta\rho_y^i)}\bar{\pi}_t \end{aligned} \tag{156}$$

The consumption function of an uninformed household, who believes $\bar{\pi}_t = \tilde{\mathbb{E}}_{t-1}^i \bar{\pi}_t$ for certain, is therefore:

$$c_t^i = \frac{1-\beta}{1-\beta\rho_y^i} y_t - \sigma\beta r_t + \frac{\beta\rho_\pi^i[(1-\beta)(\alpha^i+\lambda^i\phi^i) - \sigma(\phi^i\beta-1)(1-\beta\rho_y^i)]}{(1-\beta\rho_\pi^i)(1-\beta\rho_y^i)}\tilde{\mathbb{E}}_t^i\pi_t + \frac{\beta(1-\rho_\pi^i)[(1-\beta)(\alpha^i+\lambda^i\phi^i) - \sigma(\phi^i\beta-1)(1-\beta\rho_y^i)]}{(1-\beta)(1-\beta\rho_\pi^i)(1-\beta\rho_y^i)}\tilde{\mathbb{E}}_{t-1}^i\bar{\pi}_t \tag{157}$$

Simplifying the final two terms, we obtain equation 53.

E.2 Proof of Lemma 2 and Proposition 5

Lemma 2. The expected utility loss from imperfect information is given by:

$$\tilde{\mathbb{E}}_0^i(\hat{U}_0^{i*} - \hat{U}_0^i) = \frac{(\bar{C}^i)^{1-\frac{1}{\sigma}}}{2\sigma}\left(\left.\frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t}\right|_{\alpha_t^{i,prior}}\right)^2 \tilde{\mathbb{E}}_0^i \sum_{t=0}^{\infty} \beta^t\big((\pi_t - \tilde{\mathbb{E}}_{t-1}^i\bar{\pi}_t) - (\tilde{\mathbb{E}}_t^i\pi_t - \tilde{\mathbb{E}}_{t-1}^i\bar{\pi}_t)\big)^2 \tag{158}$$

Rewriting equation 52 with the assumption that $\bar{\pi}_t$ will remain at $\tilde{\mathbb{E}}_{t-1}^i\bar{\pi}_t$ for all t gives:

$$(\pi_t - \tilde{\mathbb{E}}_{t-1}^i\bar{\pi}_t) = \rho_\pi^i(\pi_{t-1} - \tilde{\mathbb{E}}_{t-1}^i\bar{\pi}_t) + u_{\pi t} \tag{159}$$

The information choice problem is therefore isomorphic to that in Appendix D.5, with π_t replaced with $\pi_t - \tilde{\mathbb{E}}_{t-1}^i\bar{\pi}_t$ and the constant in the objective function adjusted for $\alpha_t^{i,prior}$. The proofs in D.5 also therefore prove Lemma 2.

Proposition 5. First, define $\tilde{K}_t^i$ as the Kalman gain the household expects to use when they make their information decision (that is, assuming no updating of $\bar{\pi}_t$ beliefs). From equation 37 we have:

$$\begin{cases} \tilde{K}_t^i = 0 & \text{if } \left(\left.\frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t}\right|_{\alpha_t^{i,prior}}\right)^2 < \delta^{i*} \\ \frac{1-\tilde{K}_t^i}{(1-(\rho_\pi^i)^2(1-\tilde{K}_t^i))^2} = \frac{\delta^{i*}}{(1-(\rho_\pi^i)^2)^2}\left(\left.\frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t}\right|_{\alpha_t^{i,prior}}\right)^{-2} & \text{if } \left(\left.\frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t}\right|_{\alpha_t^{i,prior}}\right)^2 \geq \delta^{i*} \end{cases} \tag{160}$$

Among those with $\tilde{K}_t^i > 0$, we have that:

$$\frac{\partial \tilde{K}_t^i}{\partial \tilde{\mathbb{E}}_{t-1}^i\bar{\pi}_t} = \frac{\psi(1-(\rho_\pi^i)^2(1-\tilde{K}_t^i))^3}{(\Gamma_t^i)^2(1+(\rho_\pi^i)^2(1-\tilde{K}_t^i))}\frac{\partial \Gamma_t^i}{\partial \tilde{\mathbb{E}}_{t-1}^i\bar{\pi}_t} \tag{161}$$

where:

$$\Gamma_t^i = \frac{\psi(1-(\rho_\pi^2)^2)^2}{\delta^{i*}} \left(\left. \frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t} \right|_{\alpha_0^i} - \Omega^i \tilde{\mathbb{E}}_{t-1}^i \bar{\pi}_t \right)^2 \tag{162}$$

which implies:

$$\begin{aligned} \frac{\partial \Gamma_t^i}{\partial \tilde{\mathbb{E}}_{t-1}^i \bar{\pi}_t} &= -\frac{\Omega^i (\bar{C}^i)^{1-\frac{1}{\sigma}}}{\sigma} \sigma_\pi^2 \ln(2) \cdot \left(\left. \frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t} \right|_{\alpha_t^{i,prior}} \right) \\ &> 0 \text{ if and only if } \left(\left. \frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t} \right|_{\alpha_t^{i,prior}} \right) < 0 \end{aligned} \tag{163}$$

Since ρ_π^i and $\tilde{K}_t^i$ are both $\in [0,1]$, the coefficient in front of $\partial \Gamma_t^i / \partial \tilde{\mathbb{E}}_{t-1}^i \bar{\pi}_t$ in equation 161 is always positive. This proves that, for households with $\tilde{K}_t^i > 0$, and so $\sigma_{\varepsilon it}^{2*} < \infty$, $\tilde{K}_t^i$ strictly increases in $\tilde{\mathbb{E}}_{t-1}^i \bar{\pi}_t$ if and only if $\frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t}\big|_{\alpha_t^{i,prior}} < 0$. The statement in equation 61 then follows from the inverse relationship between $\sigma_{\varepsilon it}^{2*}$ and $\tilde{K}_t^i$, from the standard properties of the steady state Kalman filter. Those at the $\tilde{K}_t^i = 0$ constraint do not change attention with marginal changes in $\tilde{\mathbb{E}}_{t-1}^i \bar{\pi}_t$.

Second, we turn to the actual Kalman gains employed by the household. Define Σ as the steady state variance-covariance matrix of ξ_t conditional on the information set in period $t-1$. From the standard properties of the steady state Kalman filter:

$$\Sigma = F(\Sigma - \Sigma C(C'\Sigma C + \sigma_{\varepsilon it}^2)^{-1} C' \Sigma) F' + Q \tag{164}$$

The Kalman gain vector is then given by:

$$K_t^i = \Sigma C (C' \Sigma C + \sigma_{\varepsilon it}^2)^{-1} \tag{165}$$

The statement in equation 62 then follows from equation 61 and the fact that the elements of the Kalman gain vector grow as signal precision improves ($\partial K_t^i / \partial \sigma_{\varepsilon it}^2 < 0$).

E.3 Empirical test of Proposition 5

Proposition 5 states that information processing is increasing in perceived long-run inflation if and only if the household's subjective model has $\alpha_t^{i,prior} < 0$: that is if they start the period believing that inflation erodes real income. Since higher perceived current π_t imply higher perceived $\bar{\pi}_t$, information processing should be increasing in perceived inflation among this group. For households starting with positive models of inflation ($\alpha_t^{i,prior} > 0$), higher perceived π_t implies higher perceived $\bar{\pi}_t$, which implies less information processing. Within this group lower perceived inflation should therefore be associated with more information.

To test this, I regress perceived and expected inflation on the information indicator described in Section 3.1. For each dependent variable, I first run the regression for the households who report negative subjective models of inflation in response to Question 1, corresponding to those with $\partial c_t^i / \partial \tilde{\mathbb{E}}_t^i \pi_t < 0$. I then repeat the regression for those reporting non-negative subjective models.[39]

Table 12: Information is associated with higher perceived and expected inflation among those with negative subjective models.

	(1) Perceived	(2) Perceived	(3) Expected	(4) Expected
Information	0.226**	-0.122	0.311***	-0.0109
	(0.102)	(0.138)	(0.0990)	(0.119)
Subjective Model	Negative	Non-negative	Negative	Non-negative
Controls	Yes	Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes
R-squared	0.111	0.127	0.111	0.115
Observations	5114	2787	5298	2923

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Note: The table reports the results of regressing perceived and expected inflation on the information indicator, split by responses to Question 1. The first and third columns are the results using those who answer that inflation would make the economy weaker, and the second and fourth columns use all other respondents. All regressions are weighted using the survey weights provided in the IAS.

The results are in Table 12. Within the group with negative subjective models of inflation, both perceived and expected inflation are significantly higher among those obtaining direct information about inflation. This relationship turns negative among those with other subjective models, though this is not significant. These results are therefore in line with Proposition 5, and the model in Section 6.

E.4 Relaxing anticipated utility

In this section I relax the assumption that households make information choices assuming $\bar{\pi}_t$ will remain constant at $\partial \tilde{\mathbb{E}}_{t-1}^i \bar{\pi}_t$ for certain. Instead, they know that $\bar{\pi}_t$ follows the persistent

[39] As the information indicator is not observed every quarter there are too few observations to draw conclusions from regressions on each non-negative subjective model option individually. This is also the reason for not using the longer-horizon expectations in the IAS: the sample giving answers to both this and the information questions is small.

process:

$$\bar{\pi}_t = \bar{\rho}\bar{\pi}_{t-1} + v_t, \quad v_t \sim N(0, \sigma_v^2) \tag{166}$$

where $\bar{\rho}$ is close to but strictly less than 1. That is, I now assume that $\bar{\pi}_t$ is very persistent but stationary. This ensures that it is possible for households to pay no attention to inflation, without their utility losses from inattention becoming infinite. Note that this also implies that zero attention is no longer an absorbing state, so there is no need for the reset shocks used in Section 6.

Repeating the steps in Appendix E.1, the consumption function becomes:

$$c_t^i = \frac{1-\beta}{1-\beta\rho_y^i} y_t - \sigma\beta r_t + \frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t} \left(\tilde{\mathbb{E}}_t^i \pi_t + \frac{(1-\rho_\pi^i)\bar{\rho}}{\rho_\pi^i (1-\beta\bar{\rho})} \tilde{\mathbb{E}}_t^i \bar{\pi}_t \right) \tag{167}$$

For simplicity, I restrict households to obtaining signals of the same form as in the model without $\bar{\pi}_t$:

$$s_t^i = \pi_t + \varepsilon_t^i \quad \varepsilon_t^i \sim N(0, \sigma_{\varepsilon it}^2) \tag{168}$$

In Sections 4 and 5, this was the optimal signal structure chosen endogenously by households. This is no longer the case here. First, without this restriction households would also acquire information about $\bar{\pi}_t$ directly. Forcing households to estimate long-run inflation from realized inflation is in line with the approach taken by the literature on inflation forecasting (Stock and Watson, 2007). As $\bar{\pi}_t$ is a latent variable that cannot be observed directly in the data, it is plausible that households cannot obtain direct signals about it, but must infer it from observing other variables. Second, π_t no longer follows an AR(1) process, so unrestricted households would not choose the simple Gaussian signal over current π_t only.[40] Restricting households to the simple signal form in equation 168 is a common way to simplify rational inattention problems (e.g. Lei, 2019).

In state-space form, the subjective model is:

$$\xi_t = F^i \xi_{t-1} + e_t^i \tag{169}$$

$$s_t^i = C' \xi_t + \varepsilon_t^i \tag{170}$$

[40]It can be shown that π_t follows an ARMA(2,1) process. Even without the incentives to forecast $\bar{\pi}_t$ accurately, the optimal signal in period t would therefore also contain information on π_{t-1} and the current shock realization, as these help to forecast π_{t+1} (Maćkowiak et al., 2018).

where:

$$\xi_t = \begin{pmatrix} \pi_t \\ \bar{\pi}_t \end{pmatrix}, \quad F^i = \begin{pmatrix} \rho_\pi^i & (1-\rho_\pi^i)\bar{\rho} \\ 0 & \bar{\rho} \end{pmatrix}, \quad e_t^i = \begin{pmatrix} u_{\pi t} + (1-\rho_\pi^i)v_t \\ v_t \end{pmatrix}, \quad C = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{171}$$

It therefore remains optimal for households to incorporate signals into their perceptions of π_t and $\bar{\pi}_t$ using the Kalman filter:

$$\tilde{\mathbb{E}}_t^i \xi_t = (I - K_t^i C') F^i \tilde{\mathbb{E}}_{t-1}^i \xi_{t-1} + K_t^i s_t^i \tag{172}$$

where K_t^i is a 2×1 vector of gain parameters.

The household's attention problem is to choose the noise in their signals $\sigma_{\varepsilon it}^2$ to minimize expected utility losses from limited information plus information costs, as in Section **??**. Formally, define Σ_0 and Σ_1 as the steady state variance-covariance matrices of ξ_t conditional on the information sets in period t and $t-1$ respectively.

The per-period expected utility loss from limited information in steady state is given by:

$$\frac{(\bar{C}^i)^{1-\frac{1}{\sigma}}}{2\sigma} \left(\frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t} \right)^2 (\zeta' \Sigma_0 \zeta) \tag{173}$$

where:

$$\zeta = \left(1, \frac{(1-\rho_\pi^i)\bar{\rho}}{\rho_\pi^i (1-\beta\bar{\rho})} \right)' \tag{174}$$

Following Maćkowiak et al. (2018), the attention problem can therefore be written:

$$\min_{\sigma_{\varepsilon it}^2} \frac{(\bar{C}^i)^{1-\frac{1}{\sigma}}}{2\sigma} \left(\frac{\partial c_t^i}{\partial \tilde{\mathbb{E}}_t^i \pi_t} \right)^2 \zeta' \Sigma_0 \zeta + \frac{\psi}{2} \log_2 \left(\frac{C' \Sigma_1 C}{\sigma_{\varepsilon it}^2} + 1 \right) \tag{175}$$

Where in the steady state Kalman filter, Σ_1 and Σ_0 are defined by:

$$\Sigma_1 = F(\Sigma_1 - \Sigma_1 C (C' \Sigma_1 C + \sigma_{\varepsilon it}^2)^{-1} C' \Sigma_1) F' + Q \tag{176}$$

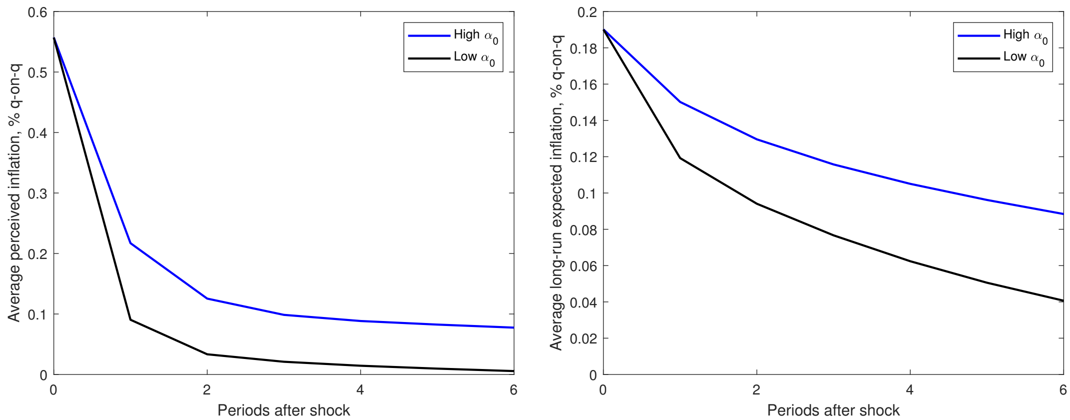
$$\Sigma_0 = \Sigma_1 - \Sigma_1 C (C' \Sigma_1 C + \sigma_{\varepsilon it}^2)^{-1} C' \Sigma_1 \tag{177}$$

And the Kalman gain vector is:

$$K_t^i = \Sigma_1 C (C' \Sigma_1 C + \sigma_{\varepsilon it}^2)^{-1} \tag{178}$$

Note that I am maintaining the assumption here that households immediately use the steady state Kalman filter each period, even though their attention is potentially changing. This is an approximation to maintain tractability, and is related to a remaining aspect of the anticipated utility assumption. Households do not expect their subjective model to change in the future, so do not expect their information processing decisions to change, even though they account for changing $\bar{\pi}_t$ in their decisions.

Figure 8: Simulated average $\tilde{\mathbb{E}}_t^i \pi_t$ and $\tilde{\mathbb{E}}_{t-1}^i \bar{\pi}_t$ for two household groups after an i.i.d. inflation shock, with time-varying $\bar{\pi}_t$ taken into account in information decision.



(a) Average $\tilde{\mathbb{E}}_t^i \pi_t$

(b) Average $\tilde{\mathbb{E}}_{t-1}^i \bar{\pi}_t$

In Figure 8 I repeat the exercise of Figure 6 above, using $\bar{\rho} = 0.99$, and adjusting ψ to 0.485×10^{-5} to ensure average $K_{1t}^i = 0.448$ before the shock hits. All other parameters are the same. The core mechanism from Section 6 remains: after the shock, low-α_0 households increase attention, and so quickly learn that inflation has fallen. High-α_0 households reduce attention, and so their perceived current and long-run inflation fall much more slowly.

F Parameters for figures in Sections 4 - 6

Figures 3 and 4:

All parameters as in Table 13, except α_0^i distributed such that $\partial c_t^i / \partial \tilde{\mathbb{E}}_t^i \pi_t$ is in the range $[-1, 1]$, and ψ set at 0.2×10^{-3}. This scales all attention so that the change in attention with subjective models is clear in the figures.

Figure 5:

The calibrated parameters are set out in Table 13. β and σ are set to standard values, and ϕ is set such that the Taylor principle is just satisfied, as found on average for the UK by Lee et al. (2013). For the remaining parameters of subjective models, including the mean of the α_0^i distribution, I estimate equations 26-28 using OLS on UK data from 1993-2019. The longer sample than the survey data is to allow for more precise estimation of model parameters. It is not extended further back because of the structural break in many UK macroeconomic time series at the end of 1992 identified by Benati (2006).

For the inflation data, I take the log first difference of quarterly CPI (ONS series MM23). I de-mean and remove seasonal variation by regressing the series on quarter-of-the-year dummies, and taking the residual as my quarterly inflation series. As well as being used in the calibration, this series is used to generate the simulated paths for perceived inflation and the aggregate consumption elasticity to inflation.

The interest rate data is 3-month money market rates, taken from the OECD Main Economic Indicators. To be consistent with the model equations, I transform this annualized rate into a gross quarterly interest rate, then take logs and de-mean. Following Harrison and Oomen (2010), I allow the mean interest rate to vary with changes in the broad regime of UK monetary policy, which I take to occur in 2009Q1 as interest rates hit the ZLB.

I proxy real income with real wages, since the model is approximated around a steady state with no saving. I begin by summing ONS series ROYH, ROYK, and ROYJ to obtain a measure of total nominal wages. I then divide this by total hours (ONS series YBUS) and working age population (ONS series MGSL) to obtain nominal wages per worker per hour. Finally, I divide by the level of CPI (including the seasonal adjustment carried out in the computation of inflation) to obtain real wages. I then take logs and hp-filter the series to obtain the cyclical component. This is estimated to be reasonably persistent, ($\rho_w = 0.731$), but still this implies a very small amplification from real income changes: $(1 - \bar{\Theta})^{-1} = 1.04$.

For $(\sigma(\alpha_0^i), \alpha_1^i, \psi)$ I target three moments from the IAS data. The first is the average ratio of 'weaker' to 'stronger' answers in response to Question 1. The raw proportions are inappropriate since we do not know how far either side of a true $dc_t^i/d\tilde{\mathbb{E}}_t^i\pi_t = 0$ is considered 'little difference' by the respondents, but the ratio still gives the balance between negative and positive models of the economy. That ratio is on average 7.533.

The second target is the estimated elasticity of the proportion with negative models to inflation, that is the coefficient from regressing Pr('weaker') on current inflation and a constant. That elasticity is 0.090.

Finally, the third target is an estimate of the average Kalman gain across the population, which helps to identify the information cost parameter ψ. For this, take Equation 30 and average across households to give:

$$\mathbb{E}_H\left(\tilde{\mathbb{E}}_t^i \pi_t\right) = \mathbb{E}_H\left(K^i\right)\pi_t + \left(1 - \mathbb{E}_H\left(K^i\right)\right)\rho_\pi \mathbb{E}_H\left(\tilde{\mathbb{E}}_{t-1}^i \pi_{t-1}\right) \tag{179}$$

where I have used the fact that all households are calibrated to have the same ρ_π, and in the model information, and so K^i, is decided before the households update their subjective models, and so is independent of perceived inflation. Denoting $\bar{\mathbb{E}}_t \pi_t$ as the average perceived inflation in time t, I therefore estimate:

$$\bar{\mathbb{E}}_t \pi_t = \gamma_1 \pi_t + \gamma_2 \bar{\mathbb{E}}_{t-1} \pi_{t-1} \tag{180}$$

by OLS, restricting $\gamma_2 = \rho_\pi(1-\gamma_1)$, where ρ_π is as in Table 13. The estimated γ_1 therefore gives an estimate of the average Kalman gain across the population. This target is 0.448.

Table 13: Calibration

Parameter	Value	Source	Parameter	Value	Source
β	0.99	standard	σ_π	0.003	estimated model
σ	1	standard	σ_i	0.004	estimated model
ϕ	β^{-1}	Lee et al. (2013)	σ_y	0.008	estimated model
$\mathbb{E}_H \alpha_0^i$	-0.732	estimated model	$\sigma(\alpha_0^i)$	0.613	targets
λ	-0.037	estimated model	α_1^i	-234	targets
ρ_π	0.329	estimated model	ψ	0.787×10^{-9}	targets
ρ_y	0.731	estimated model			

Figures 6 and 7:

All shared parameters are as in Table 13, except for ψ, which is set to 0.453×10^{-9} to ensure that average K_{1t}^i remains equal to the target level from the survey (0.448) in the period before the shock. For Figure 6, the high-α group have $\alpha_0^i = 0.997$, while the low-α group have $\alpha_0^i = -0.923$. These are chosen such that both households have the same initial $K_{1t}^i = 0.7$. The variance of v_t in equation 56 is set at $\sigma_v^2 = \sigma_\pi^2/10$, and the reset shock probability is set at 0.005.

To simulate these figures, optimal attention is derived using equation 160. The variance of noise in the signals is then given by:

$$\sigma_{\varepsilon it}^2 = \frac{\sigma_\pi^2(1 - \tilde{K}_t^i)}{\tilde{K}_t^i(1 - (\rho_\pi^i)^2(1 - \tilde{K}_t^i))} \tag{181}$$

Plugging this into equations 164 and 165 for each household each period gives the Kalman gain vector, to be used to simulate the path of each household's expectations.