Heterogeneity in imperfect inflation expectations: theory and evidence from a novel survey*

Alistair Macaulay† James Moberly‡

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Abstract

Using novel survey data from Germany, we study heterogeneity in how households form inflation expectations. We elicit (i) uncertainty in perceptions of current inflation, and (ii) how persistent households perceive inflation to be. Combining these with standard survey questions on inflation, we infer laws of motion for expectations at the individual level. Based on averages alone, a standard model calibrated to our data predicts inflation shocks generate small and transitory responses in expectations and consumption. The considerable heterogeneity we observe in expectation formation, however, amplifies the transmission to aggregate consumption by an order of magnitude, and substantially increases its persistence.

JEL codes: D83, D84, E31, E71

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†School of Economics, University of Surrey. Email: a.macaulay@surrey.ac.uk

‡Department of Economics & University College, University of Oxford. Email: james.moberly@economics.ox.ac.uk

1 Introduction

Households' information and subjective models of inflation shape how their inflation expectations respond to macroeconomic shocks. However, commonly used survey data on expectations is consistent with various combinations of information and subjective models, with contrasting implications. The well-known regressions in Coibion and Gorodnichenko (2015a), for example, are consistent with models in which information is noisy but households know the true law of motion for inflation (Coibion and Gorodnichenko, 2015a), or models with full information but misspecified forecasting rules (Gabaix, 2020; Hajdini, 2020). How then do households form their expectations?

This paper uses novel survey data to answer that question. We add new questions to the Bundesbank's Survey on Consumer Expectations to elicit the uncertainty in household perceptions of current inflation, and how persistent households perceive inflation to be. When combined with responses to other standard survey questions, this allows us to infer subjective laws of motion and details of information processing at the individual level.

We find that, on average, uncertainty about current inflation is low, and the perceived persistence of inflation is close to that of realized inflation. However, these averages mask considerable heterogeneity, which is important for understanding aggregate behavior. Calibrating a standard consumption-saving model to our data, heterogeneity in the expectations process amplifies the aggregate consumption response to inflation shocks by an order of magnitude, relative to the representative-agent case. Expectations may therefore play a substantially larger role in business cycle fluctuations than implied by models based on aggregate expectations alone.[1]

To see how these questions can inform models of expectation formation, suppose that each household i believes inflation follows the AR(1) process:[2]

$$\begin{aligned} \pi_{t+1} &= \tilde{\rho}_i \pi_t + \varepsilon_{t+1} \\ \varepsilon_{t+1} &\sim N(0, \tilde{\sigma}^2_{\varepsilon,i}) \end{aligned} \tag{1}$$

Uncertainty in the inflation forecast can then be decomposed into: (i) uncertainty about current inflation, and (ii) uncertainty arising from future shocks:[3]

$$\tilde{Var}_{i,t}(\pi_{t+1}) = \tilde{\rho}_i^2 \tilde{Var}_{i,t}(\pi_t) + \tilde{\sigma}^2_{\varepsilon,i} \tag{2}$$

[1] e.g. Fuster et al. (2010), Bhandari et al. (2019), Angeletos et al. (2020), and many others.

[2] Numerous studies suggest household forecasts are well-characterized by such simple forecasting rules (e.g. Adam, 2007). We relax this assumption in Appendix A.

[3] Note this abstracts from uncertainty about $\tilde{\rho}_i$, as is common in models with parameter learning (see e.g. Bullard and Suda, 2016).

Of the terms in this equation, existing survey questions only measure subjective uncertainty in the forecast $\tilde{Var}_{i,t}(\pi_{t+1})$. However, different combinations of the two components of this uncertainty imply very different expectation dynamics, for the same observed forecast uncertainty. To illustrate, consider three ways of decomposing a given $\tilde{Var}_{i,t}(\pi_{t+1})$, assuming that the true law of motion for inflation is persistent but stationary:

1. $\tilde{Var}_{i,t}(\pi_t) = 0$, $\tilde{\rho}_i = 1$, $\tilde{\sigma}^2_{\varepsilon,i} = \tilde{Var}_{i,t}(\pi_{t+1})$. Agents have full information about current inflation, but over-extrapolate from current realizations to forecasts of future inflation. Expectations overshoot the true path of π_t.
2. $\tilde{Var}_{i,t}(\pi_t) = \tilde{Var}_{i,t}(\pi_{t+1})$, $\tilde{\rho}_i = 1$, $\tilde{\sigma}^2_{\varepsilon,i} = 0$. Agents are uncertain about current inflation, implying they have imperfect information. Inflation perceptions therefore respond slowly to shocks. The high $\tilde{\rho}_i$ implies agents still overextrapolate from perceptions to forecasts, generating 'delayed overshooting' (Angeletos et al., 2020).
3. $\tilde{Var}_{i,t}(\pi_t)$ equal to any real number, $\tilde{\rho}_i = 0$, $\tilde{\sigma}^2_{\varepsilon,i} = \tilde{Var}_{i,t}(\pi_{t+1})$. Agents underextrapolate. Forecasts do not respond to current inflation.

A given level of uncertainty in inflation forecasts could be consistent with a continuum of models between these extremes. Our questions pin down the correct model at the household level by eliciting the variance of the inflation perception $\tilde{Var}_{i,t}(\pi_t)$, and the perceived persistence $\tilde{\rho}_i$. Combined with existing questions eliciting $\tilde{Var}_{i,t}(\pi_{t+1})$, we then use equation 2 to infer the perceived variance of the innovations $\tilde{\sigma}^2_{\varepsilon,i}$.

To measure uncertainty in perceptions, we start with an existing survey question eliciting a point estimate of the respondent's inflation perception. We then add a new question, asking for the probability that the inflation rate lies within a specified range around that point estimate. Fitting a triangular distribution to these responses yields an estimate of $\tilde{Var}_{i,t}(\pi_t)$. To measure $\tilde{\rho}_i$, we present respondents with hypothetical scenarios of macroeconomic shocks, as in Andre et al. (2022). We specify that the shock has increased current inflation by one percentage point, and ask how the respondent would update their inflation expectations as a result. These novel questions are critical to answer our question: even with panel data on $\tilde{Var}_{i,t}(\pi_{t+1})$, as in the US Survey of Consumer Expectations, it is not possible to jointly identify $\tilde{Var}_{i,t}(\pi_t)$, $\tilde{Var}_{i,t}(\pi_{t+1})$, and $\tilde{\rho}_i$ at the individual level without adding further substantial restrictions to models of expectation formation.

Our finding that, on average, respondents are less uncertain about current inflation than future inflation suggests that much of the uncertainty in expectations comes from perceived noise in the inflation process, not a lack of information about current inflation. However, a minority of households are very uncertain in their inflation perceptions. Perceived persistence is similarly heterogeneous: while two-thirds of households perceive no inflation persistence at the one-year horizon, those who do update their expectations after the hypothetical shock often do so by a large amount.

This heterogeneity has a large impact on the dynamics of aggregate consumption. In a standard consumption-saving model, the individual consumption response to inflation is convex in perceived inflation persistence. The same path of aggregate inflation expectations is therefore associated with much larger fluctuations in aggregate consumption if the individual-level subjective laws of motion are heterogeneous. The response of aggregate consumption to an inflation shock is consequently an order of magnitude larger when we calibrate the model using the heterogeneity observed in our data, relative to a representative-agent calibration based on average parameters. The persistence of aggregate consumption also rises by nearly one-half.

Moreover, elements of household expectation formation correlate systematically with each other, and with household characteristics, further distorting aggregate consumption away from the representative-agent case. In particular, hand-to-mouth households with little liquid wealth are more uncertain about future inflation relative to current inflation, and believe inflation is substantially more persistent. Since these households are likely to be less responsive to changes in their expectations, this dimension of heterogeneity further underlines the importance of investigating beyond average parameters of expectation formation for understanding macroeconomic behavior.

Related Literature. Angeletos et al. (2020) also study inflation expectations with imperfect information and possible misperceptions of inflation persistence, by directly estimating impulse responses of average expectations to shocks. Our approach is complementary. While they can capture richer subjective models than the linear approximations we identify, our approach reveals heterogeneity in expectation formation, which cannot be observed with average forecasts. Similarly, Ryngaert (2018) estimates perceived persistence and signal precision among professional forecasters, but cannot uncover heterogeneity in these parameters. Binder et al. (2022) show that such heterogeneity is necessary to account for several stylized facts in survey data on inflation uncertainty among professional forecasters, but are not able to measure the relevant parameter distributions.

There are large literatures measuring noisy information and perceived persistence separately, using data from surveys (e.g. Coibion and Gorodnichenko, 2012, 2015a; Laudenbach et al., 2021; Link et al., 2023; Kikuchi and Nakazono, 2023) and lab experiments (e.g. Adam, 2007; Beshears et al., 2013; Caplin and Dean, 2015; Afrouzi et al., 2023). Consistent with our results, these papers frequently find substantial heterogeneity in the aspect of expectation formation they study.[4] Other studies document large heterogeneity

[4]Coibion and Gorodnichenko (2012, 2015a) find no evidence of heterogeneous signal-to-noise ratios. However, their tests are derived under the assumption that forecasters believe inflation follows a random walk, contrary to the low average perceived persistence in our sample, and in other surveys (Jain, 2017; Ryngaert, 2018).

in uncertainty over the future of a range of macroeconomic variables (see Potter et al., 2017, for a survey). We contribute to these literatures by directly measuring the perceived persistence of inflation simultaneously with variance in inflation perceptions and expectations, all at the individual level. While previous studies typically infer at least one of these objects from assumptions on information or subjective laws of motion, we show that identifying them separately is important to understand the dynamics of aggregate expectations and consumption. In particular, to the best of our knowledge there are no existing quantitative measures of the variance in inflation perceptions.[5]

In analyzing these distributions, we also contribute to the literature on the role of expectations in business cycles. Like us, Branch and Evans (2006), Hommes and Lustenhouwer (2019), Macaulay (2022), Pedemonte et al. (2023) (among others) find that heterogeneity in the expectation formation process can substantially alter macroeconomic outcomes. We directly measure the relevant heterogeneity, and show that the resulting distribution of expectation processes amplifies the aggregate consumption response to inflation by an order of magnitude.

Our approach is not the only possible way to capture the effects of heterogeneity in expectation formation we identify. While data on market-based uncertainty (Bauer et al., 2022) or disagreement between agents (Mankiw et al., 2004; Andrade et al., 2016) cannot identify individual-level properties of expectation formation, with further structural assumptions it may be possible to use questions in existing individual-level surveys to estimate our parameters. We are not aware of any studies currently taking this approach for both information and subjective laws of motion simultaneously. For example, Jain (2017) estimates perceived inflation persistence using the panel dimension of the US Survey of Professional Forecasters, but does not disentangle the uncertainty over current and future inflation. To extend that work, one possibility would be to make use of density forecasts of inflation over very short horizons, which could be used to approximate the uncertainty over current inflation we measure for households.

We pursue our approach over such a method for two reasons. First, short-horizon density forecasts are typically only available for professional forecasters, who are substantially better-informed about current economic events than households (Link et al., 2023). Second, our direct measurements allow us to document some of the key properties of expectation heterogeneity with less restrictive structural assumptions. For example, methods inferring perceived persistence by comparing expectations at different horizons typically assume households hold identical long-run expectations (Reis, 2020; Andre et al., 2021), and that there is no 'forward information' about future inflation in-

[5] Armona et al. (2019) ask households to rate their uncertainty over past house-price growth on a 1-5 scale, but do not relate this quantitatively to the variance in expectations. The uncertainty over *future* inflation elicited in existing surveys is qualitatively different, as that inflation is not yet realized.

novations (Goldstein and Gorodnichenko, 2022). Our method is robust to relaxing both of these assumptions (Appendix A.2), and with no panel dimension we also avoid survey tenure effects (Crossley et al., 2017), which can be large for questions on inflation (Kim and Binder, 2022).

2 Expectations Framework

2.1 The Agent

Each agent i believes inflation follows the AR(1) process in equation 1. This subjective law of motion for inflation may or may not coincide with the true data generating process.

Agents never observe current or past inflation directly. Rather, each period agent i receives a noisy signal $s_{i,t}$ about current inflation:

$$\begin{aligned} s_{i,t} &= \pi_t + q_{i,t} \\ q_{i,t} &\sim N(0, \sigma_q^2) \end{aligned} \tag{3}$$

Agents perceive the variance of $q_{i,t}$ to be $\tilde{\sigma}_{q,i}^2$, which is not necessarily equal to σ_q^2 (as in e.g. Broer and Kohlhas, 2019). This allows the model to be consistent with any survey respondents who are simultaneously incorrect, but very certain, about their inflation perception.

After observing the signal, agents update their perception of inflation using the steady-state Kalman filter, then use that to form forecasts of future inflation.[6] The posterior one-period ahead inflation forecast is:

$$\tilde{E}_{i,t}\pi_t = (1 - \chi_i)\tilde{E}_{i,t-1}\pi_t + \chi_i s_{i,t} \tag{4}$$

$$\tilde{E}_{i,t}\pi_{t+1} = \tilde{\rho}_i \tilde{E}_{i,t}\pi_t \tag{5}$$

$$\chi_i = 1 - \frac{V_i^p}{V_i^f} \tag{6}$$

where the operator $\tilde{E}_{i,t}$ denotes the expectation of agent i in period t, and χ_i is agent i's Kalman gain. V_i^p and V_i^f denote respectively the steady-state subjective variances of perceived inflation ($\tilde{Var}_{i,t}(\pi_t)$) and one-period ahead inflation ($\tilde{Var}_{i,t}(\pi_{t+1})$). These are such that $V_i^f \geq V_i^p$. All derivations for results here, and in Section 5, are in Appendix A.

The formula for χ_i is intuitive: if V_i^p is low relative to V_i^f, signals on current inflation

[6] The assumption of steady-state filtering is required to identify the Kalman gain in the absence of panel data. The same assumption is commonly made in the rational inattention literature for tractability (Maćkowiak and Wiederholt, 2009).

must be informative, and much of the uncertainty in future inflation must be due to future shocks. With informative signals, agents optimally make large adjustments to their inflation perceptions and forecasts in light of signal realizations. In the extreme with full information ($\sigma_q^2 = \tilde{\sigma}_{q,i}^2 = 0$), agents observe current inflation without noise, and so $V_i^p = 0$ and $\chi_i = 1$: expectations depend entirely on signals, and not on prior beliefs formed in previous periods. Note that the forecast variance in a given period, as measured in several existing surveys, does not place any restrictions on the Kalman gain. This is why we require a novel question, not present in existing surveys, to identify V_i^p separately to V_i^f.

Using equation 1, $\tilde{\sigma}_{\varepsilon,i}^2$ is given by:

$$\tilde{\sigma}_{\varepsilon,i}^2 = V_i^f - \tilde{\rho}_i^2 V_i^p \tag{7}$$

Finally, the perceived signal noise is:

$$\tilde{\sigma}_{q,i}^2 = \frac{V_i^f V_i^p}{V_i^f - V_i^p} \tag{8}$$

Our novel questions measuring V_i^p and $\tilde{\rho}_i$ therefore allow us to infer all parameters of the law of motion for inflation expectations (equations 4-6), and the variances of both fundamental shocks and signal noise.

Of course, interpreting our data in this way assumes that an AR(1) process is a good description of household beliefs about the law of motion for inflation, as documented in e.g. Adam (2007); Goldstein and Gorodnichenko (2022). Our key results, however, are robust to relaxing this assumption. In Appendix A.2 we extend the model to a richer set of subjective laws of motion, which may include other variables (e.g. output, interest rates), longer lags, and heterogeneous long-run expectations. Equations 6 and 8 still capture household i's Kalman gain and perceived noise variance respectively, and all the transmission channels discussed below continue to operate. The main effect of relaxing the AR(1) assumption is that equation 7 no longer captures the perceived variance of inflation shocks, but rather gives a composite of all sources of uncertainty that are not related to current inflation.

2.2 Expectations Impulse Responses

For simplicity, assume inflation is indeed an exogenous AR(1) process, with true autocorrelation ρ:

$$\pi_t = \rho \pi_{t-1} + \varepsilon_t \tag{9}$$

We consider the impulse response of expectations to a one percentage-point shock to inflation at $t = 0$, with inflation and inflation expectations at steady-state (zero) before the shock. Abstracting from the effect of realized signal noise $q_{i,t}$,[7] the one-period ahead inflation forecast of agent i, t periods after the shock, is:

$$\tilde{E}_{i,t}\pi_{t+1} = \tilde{\rho}_i\chi_i \frac{\rho^{t+1} - (1-\chi_i)^{t+1}\tilde{\rho}_i^{t+1}}{\rho - (1-\chi_i)\tilde{\rho}_i} \tag{10}$$

Different combinations of χ_i and $\tilde{\rho}_i$ therefore imply very different impulse responses of expectations, even for the same V_i^f. On impact, the response of expectations is increasing in χ_i and $\tilde{\rho}_i$. The persistence of the expectation response increases in $\tilde{\rho}_i$, but decreases in χ_i. If $\tilde{\rho}_i$ is sufficiently large, and χ_i sufficiently small, then expectations display hump-shaped impulse responses (as observed in e.g. Angeletos et al., 2020). Equally, as in Angeletos et al. (2020), if $\tilde{\rho}_i > \rho$ then expectations overshoot, rising above realized inflation some periods after the shock.

2.3 The Role of Heterogeneity

If agents were homogeneous, equation 10 would also describe the impulse response of aggregate inflation expectations to the shock. With heterogeneity in $\tilde{\rho}_i$ and χ_i, however, the initial response of aggregate inflation expectations to the shock becomes:

$$\tilde{E}_0\pi_1 = E[\tilde{\rho}_i]E[\chi_i] + Cov(\tilde{\rho}_i, \chi_i) \tag{11}$$

A positive correlation between $\tilde{\rho}_i$ and χ_i therefore amplifies the initial effect of the shock on expectations, because those who extrapolate the most from perceived to expected inflation also update their perceptions the most in the period of the shock.

Heterogeneity continues to affect aggregate expectations in the periods after the shock. In Appendix A, we show that heterogeneity in $\tilde{\rho}_i$ increases the persistence of the response of aggregate expectations to the shock.

3 Data

We use the November 2021 wave of the Bundesbank-Online-Panel-Households survey, which is administered online to a representative sample of the German population. 4110 households were asked our questions.

In the main survey, households give a point estimate of the inflation rate over the past 12 months, and give both point and density forecasts of inflation over the next 12 months.

[7] This captures the average expectation across many agents who share the same $\tilde{\rho}_i$ and χ_i.

The density forecast involves households filling out the probabilities of inflation falling within various intervals, as in other recent household surveys (Armantier et al., 2017; Coibion et al., 2021). Additionally, a range of household characteristics are collected. We report summary statistics in Appendix B.

We add two novel questions for the November 2021 wave, reproduced in Table 1 (see Appendix B for the German translations seen by respondents, and the point and density forecast questions).

Question 1 elicits the uncertainty in the household's perceptions of current inflation.[8] The high and low inflation values seen by the respondent are their point estimate of current inflation, ±1 percentage-point. If the respondent's point inflation estimate is ≥5%, this range is widened to ±2 percentage-points, as uncertainty in inflation expectations is known to rise with point estimates (De Bruin et al., 2011; Ben-David et al., 2018). Answers are in percent, and must be within $[0, 100]$. Respondents also see a note giving further explanation of the question (see Appendix B).

To calculate the variance of perceived inflation V_i^p, we fit a symmetric triangular distribution using the respondent's answer and their point estimate, and then take the variance of the fitted distribution. This is similar to the approach in Coibion et al. (2021), and in the Survey of Consumer Expectations when respondents only report positive probabilities in two bins of a density forecast question (Armantier et al., 2017). Full details of this, and the construction of all other variables, are in Appendix C.1.

In computing the Kalman gain for each respondent, we take the ratio of V_i^p to V_i^f, the variance of expected inflation (equation 6). We measure V_i^f by fitting a piecewise-linear distribution to the subjective probabilities given by households for ranges of future inflation, and calculating the variance of that distribution (see Appendix C.1). This strikes a balance between limiting the discrepancy between the methods for computing V_i^p and V_i^f, while still making use of all the information in each household's density forecast.[9] To confirm that the difference in measurement approaches does not bias our results, in Appendix C.2 we construct an alternative measure of V_i^f, which utilizes less of the available information from the density forecasts, but which corresponds closely to the measurement of V_i^p. All results below are robust to this alternative. This is consistent with the hypothesis in Kumar et al. (2022) that the discrepancy they find between their triangular and density variance measures is driven by a difference in the treatment of the

[8]We do not use a multiple-bin density forecast, as for inflation expectations, as these questions are cognitively demanding. Previous waves of this survey have found that including too many can result in households dropping out of the survey.

[9]Other approaches to estimating uncertainty from density forecasts such as fitting generalized-beta distributions (Engelberg et al., 2009) or employing non-parametric methods (Del Negro et al., 2022) have several attractive properties, but would imply greater methodological differences between V_i^f and V_i^p.

Table 1: Novel questions added to the BOP-HH survey in November 2021

Question	Text	Sample
1	Now we would like to know how certain you are about your information on the inflation rate or deflation rate over the past 12 months ([Value of point estimate])%. In your opinion, how likely is it that the inflation rate has been between [Low inflation level]% and [High inflation level]% over the past twelve months?	All respondents
Respondents randomly shown one of three scenarios before Question 2		
General	Imagine the following hypothetical situation: Due to an unexpected economic event, the inflation rate increased by one percentage point in the past year.	Group A
Supply	Imagine the following hypothetical situation: Due to unexpected problems with local production technology in the Middle East, the price of crude oil rose in the past year, causing the inflation rate to rise by one percentage point.	Group B
Demand	Imagine the following hypothetical situation: Due to increased defense spending, government spending rose unexpectedly more than usual in the past year, causing the inflation rate to rise by one percentage point. The change is temporary and occurs even though the government's assessment of national security or economic conditions has not changed. In addition, taxes do not change in response to the spending program.	Group C
2	In this situation, would you adjust your inflation expectations for the next 12 months as stated in the first part of the questionnaire? If so, to what extent?	All respondents

end-points of each distribution, which is not present here.[10]

Question 2 elicits perceived inflation persistence $\tilde{\rho}_i$. Following Andre et al. (2022), respondents are given a hypothetical scenario describing an exogenous shock, and asked how they would expect that to affect future inflation. Unlike Andre et al. (2022), in each scenario we tell the respondents that the shock caused current inflation to increase by 1 percentage-point. Their answers on how that would change their inflation expectations consequently reflect their estimates of inflation persistence, not their predictions of the immediate impact of the shock, which Andre et al. (2022) find to be heterogeneous across

[10]Specifically, Kumar et al. (2022) fit a triangular distribution using questions about firms' most optimistic, and most pessimistic, growth expectations. As they use these as the end-points of the distribution, they are assigned zero probability mass. In contrast, their density forecast question allows firms to place positive probability mass on these highest and lowest forecasts. Our measure of V_i^p does not involve elicited end-points, so we avoid this.

households.

In Section 2 we did not distinguish between different types of shocks, and our main empirical analysis will do the same. However, households may associate different shocks with different levels of persistence. To investigate this, we randomly split respondents into three groups. The first group are not told the nature of the shock, the second see a hypothetical supply shock (oil price), and the third see a demand shock (government spending). The specific scenarios are adapted from Andre et al. (2022).

To answer, respondents see:

1) Yes, from [Value of point estimate]% to __%
2) No

and either input a number in the first line or select 'No'.

As we tell respondents the precise size of the hypothetical innovation to inflation in each scenario, we interpret any predicted change in future inflation as a direct measure of perceived persistence. In the language of the model, we assume the scenarios increase hypothetical $\tilde{E}_{i,t}\pi_t$ by 1 percentage-point, and so from equation 5 the resulting change in $\tilde{E}_{i,t}\pi_{t+1}$ is equal to $\tilde{\rho}_i$.

Using these novel questions we therefore obtain $\tilde{\rho}_i$ and V_i^p for each respondent. We then infer the implied χ_i, $\tilde{\sigma}^2_{\varepsilon,i}$, and $\tilde{\sigma}^2_{q,i}$ using equations 6-8.

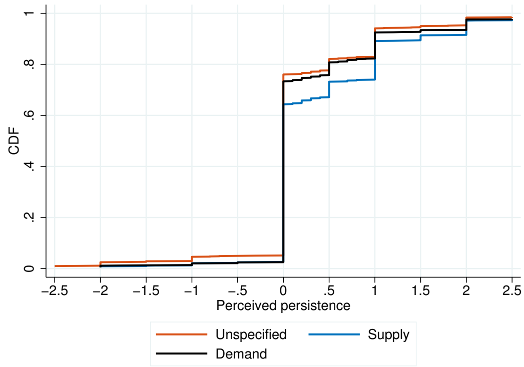
As our questions were included in the November 2021 wave of the survey, the results below are influenced by the inflation context in that period. Year-on-year CPI inflation in Germany was close to 5%, having risen from 1% in January 2021. Consistent with the evidence that attention to inflation rises in such episodes (Weber et al., 2023), in Appendix D we present data from Google Trends that shows internet searches in Germany for "inflation" rose substantially from January to November 2021. Our results are therefore most applicable to environments with high inflation. However, November 2021 is not unique in this respect: inflation continued to rise for more than a year afterwards, and Google searches only fell back below their November 2021 level in April 2023. Furthermore, if greater salience of inflation is associated with more volatile inflation expectations (as found in e.g. Pfäuti, 2023), then understanding the determinants of expectations is particularly important in high inflation environments like that of November 2021.

4 Empirical Results

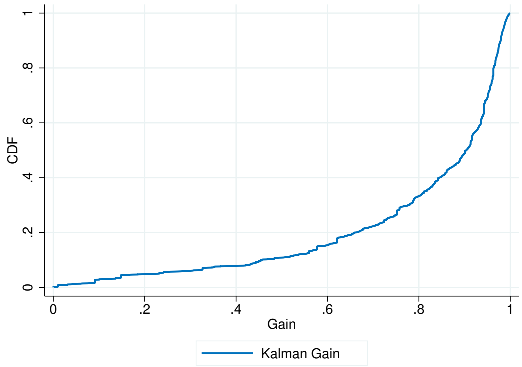
Figure 1a plots the CDF of $\tilde{\rho}_i$, truncated to remove the approximately 1% of responses outside $[-5, 5]$. Of the remaining responses, 89% report $\tilde{\rho}_i \in [0, 1]$, and 68% do not revise their expectations at all.

Result 1 *Conditional on perceiving persistence in* $[0,1]$*, the average perceived persistence is broadly consistent with the data. However, the cross-sectional heterogeneity is large.*

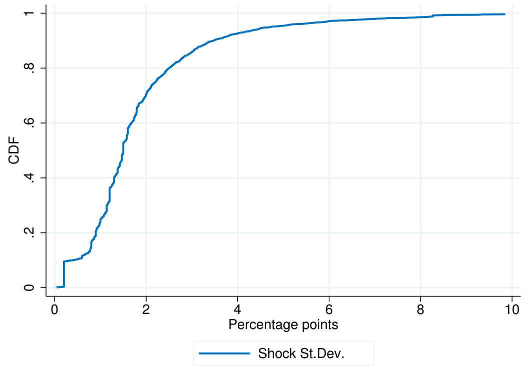
Of those with $\tilde{\rho}_i \in [0,1]$, the mean $\tilde{\rho}_i$ is 0.18, close to the 'correct' answer of 0.21 based on recent German data.[11] Including all responses in $[-5,5]$, the mean $\tilde{\rho}_i$ is 0.29. The heterogeneity, however, is large. Restricting to responses in [0,1], the maximum possible standard deviation is 0.5: the standard deviation in our sample is 0.36.



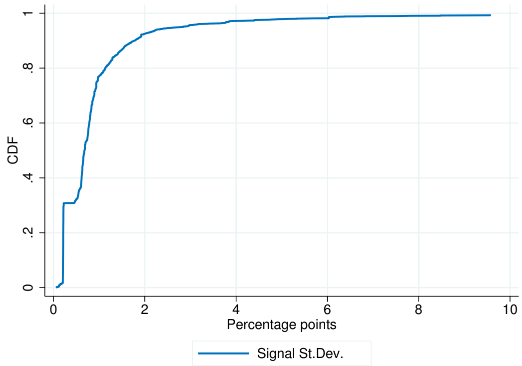
(a) CDF of $\tilde{\rho}_i$



(b) CDF of χ_i



(c) CDF of $\tilde{\sigma}_{\varepsilon,i}$



(d) CDF of $\tilde{\sigma}_{q,i}$

Figure 1: CDFs of key parameters. Source: Bundesbank-Online-Panel-Households, November 2021 wave.

In Appendix D we show that this heterogeneity is not driven by households rounding to the nearest percentage-point when reporting expectations (as studied in Binder, 2017). We continue with the full sample here, as rounded expectations may still matter for consumption decisions. The large fraction of households choosing not to update expectations is consistent with other information treatments in previous waves of this survey (Dräger et al., 2022), and with other hypothetical scenario surveys (Christelis et al., 2021; Fuster et al., 2021).

[11]This is the coefficient from a linear projection of annual CPI inflation on its lagged value (data from *www.destatis.de*, 2002-2021).

Splitting by the shock type presented to respondents, the mean $\tilde{\rho}_i$ measurements for those within the $[0, 1]$ interval are 0.16 (unspecified shock), 0.22 (supply shock), and 0.16 (demand shock). Supply shocks are therefore perceived to be slightly more persistent than demand shocks, consistent with evidence that supply shocks are particularly important for household inflation expectations (Coibion and Gorodnichenko, 2015b). While some of this difference could stem from the stronger reference to the temporary nature of the demand shock in our scenarios, note that the unspecified scenario is very similar to the supply shock in this respect; neither state explicitly that the shock is temporary. The fact that average $\tilde{\rho}_i$ is higher for supply shocks than *both* alternatives supports the interpretation that supply shocks are perceived to be more persistent.

Figure 1b shows the CDF of χ_i.[12]

Result 2 *The average Kalman gain is high, at 0.8. There is considerable cross-sectional heterogeneity.*

The high average Kalman gain stems from most consumers being considerably more certain about their inflation perceptions than their expectations. There is, however, a long tail of very uncertain households, with low Kalman gains.

The average χ_i exceeds values obtained from regressions on average forecast errors, which are typically close to 0.5 (e.g. Coibion and Gorodnichenko, 2015a). The discrepancy is unsurprising, since such regressions yield biased estimates if agents hold inaccurate beliefs about inflation persistence (Ryngaert, 2018). In addition, consumers were plausibly better-informed about inflation than in other periods because inflation was rising, and was subject to elevated media coverage.

Figure 1c shows the CDF of $\tilde{\sigma}_{\varepsilon,i}$. There is considerable heterogeneity; a tail of households believe future inflation is extremely volatile. This reinforces that much of the uncertainty in inflation expectations relates to future shocks, rather than uncertainty about current inflation. Finally, Figure 1d shows the CDF of $\tilde{\sigma}_{q,i}$. Reflecting the high average χ_i, most households have little noise in their signals, though a minority have very imprecise information. Further distributions, including those of V_i^p and V_i^f used to calculate χ_i, and the relationships of these variables with point inflation forecasts, are presented in Appendix D.

[12]For households who are completely certain that π_t is within the interval shown in our question, V_i^p cannot be point-identified, so we obtain a range of possible values for χ_i (details in Appendix C.1). Figure 1b uses the mid-point in these cases, dropping respondents with a range of width >0.2. The CDFs using the upper and lower bounds on χ_i are shown in Appendix D. Figures 1c and 1d similarly use midpoints of implied ranges in these cases.

4.1 Relationships between Expectation Components

Table 2 shows our next main result:

Result 3 *Households who are more uncertain about current inflation are also more uncertain about future inflation, believe inflation shocks are more volatile, and have lower Kalman gains.*

Table 2: Cross-sectional correlations of subjective law of motion elements.

	$SD_i(\pi_{t+1})$	$SD_i(\pi_t)$	$\tilde{\rho}_i$	$SD_i(\varepsilon_{t+1})$	χ_i
$SD_i(\pi_{t+1})$	1.000				
$SD_i(\pi_t)$	0.476***	1.000			
$\tilde{\rho}_i$	0.036*	-0.015	1.000		
$SD_i(\varepsilon_{t+1})$	0.988***	0.443***	-0.034	1.000	
χ_i	0.316***	-0.393***	0.038*	0.337***	1.000

Note: Bundesbank-Online-Panel-Households, November 2021 wave. For cases where χ_i is set-identified, respondents are excluded if the parameters are estimated very imprecisely (range>0.2). For all remaining set-identified parameters, the mid-point of the range is used. Observations of $SD_i(\pi_{t+1})$, and $SD_i(\pi_t)$ below the 1st or above the 99th percentile of that variable's distribution are also excluded as outliers, as are observations of $\tilde{\rho}_i$ outside $[-5, 5]$ (c.1% of observations). * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

This is consistent with noisy information models, in which greater uncertainty arises when households process less information. Importantly, this result is not imposed by our structural assumptions. Our model allows more uncertainty about current inflation to be associated with higher or lower Kalman gains, depending on the relationships between V_i^p and V_i^f (equation 6). Indeed, noisy information can only partly explain the distribution of uncertainty in the data. More uncertain households also believe that inflation shocks are more volatile (higher $\tilde{\sigma}_{\varepsilon,i}$). This further highlights the importance of measuring the uncertainty in perceptions and expectations separately.

There are small positive correlations of $\tilde{\rho}_i$ with uncertainty about future inflation, and with Kalman gains. However, in Appendix D we break this down, and find that households who believe inflation is non-stationary ($|\tilde{\rho}_i| \geq 1$) are qualitatively different from households with $|\tilde{\rho}_i| < 1$. Within households who believe inflation is persistent and stationary, greater perceived persistence is associated with less uncertainty about current and future inflation, less perceived noise in the inflation process, and greater Kalman gains. That is consistent with models of endogenous information acquisition; if inflation is more persistent, information about current inflation is more valuable.[13] We also split the sample by shock scenario, but find little difference across shock types.

[13]Note the theoretical literature has also identified other channels through which persistence affects

4.2 Correlations with Household Characteristics

As our application in Section 5.2 is to consumption, Table 3 shows results from regressing each component of expectation formation on household characteristics known to relate to Marginal Propensities to Consume (MPCs). The key variables are liquid wealth (bank deposits plus securities), illiquid wealth (property plus firm ownership), other wealth, debt, and household income. There is also an indicator for if the household is hand-to-mouth, defined here as having liquid wealth of less than €1250.

Since households with $\tilde{\rho}_i = 0$ may differ qualitatively from those with $\tilde{\rho}_i \neq 0$, the final column restricts the sample to those with $\tilde{\rho}_i \neq 0$. This gives the estimated associations conditional on the household revising expectations in light of the hypothetical shock.[14] In Appendix D we show that selection into $\tilde{\rho}_i \neq 0$ is not significantly related to wealth or income.

The first row of coefficients shows our next main result:

Result 4 *Hand-to-mouth households are more uncertain about future inflation, but no more uncertain about current inflation, than other households. They believe inflation is noisier and more persistent, and have 8.6% higher Kalman gains on average.*

Angeletos et al. (2020) find that aggregate inflation expectations display delayed overshooting, suggesting households perceive inflation to be more persistent than implied by the true data-generating process. Our results suggest that aggregate over-persistence may partly reflect the expectations of hand-to-mouth households, who are less able to respond to expected inflation by adjusting consumption and saving. Their expectations may not therefore have much impact on aggregate dynamics. While high Kalman gains for hand-to-mouth households is inconsistent with simple models of rational inattention, this result could be driven by those who are close to leaving their borrowing constraints, who have highly non-linear policy functions and value information as a result (Broer et al., 2021).

Above the hand-to-mouth threshold, higher liquid wealth has little relationship with expectation formation. Higher liquid wealth is associated with a statistically significant increase in the Kalman gain, but this is quantitatively small: each further €1000 is associated with a 0.02% rise in χ_i.

The final row of coefficients gives our next main result:

optimal information choices Maćkowiak and Wiederholt (2009). The overall relationship between persistence and information choices may therefore be highly nonlinear in theory, just as it is in the survey.

[14] This is equivalent to the linear component of a hurdle model in which, unlike Cragg (1971), we do not impose that $\tilde{\rho}_i$ is truncated to be ≥ 0.

Table 3: Regressions of components of subjective laws of motion on household characteristics.

	(1) $\log(SD_i(\pi_{t+1}))$	(2) $\log(SD_i(\pi_t))$	(3) $\log(SD_i(\varepsilon_{t+1}))$	(4) $\log(\chi_i)$	(5) $\tilde{\rho}_i$
Hand-to-mouth	0.1407***	0.0296	0.1518***	0.0859**	0.2716**
	(0.0451)	(0.0384)	(0.0484)	(0.0432)	(0.1326)
Liquid wealth	0.0001	-0.0001	0.0001*	0.0002***	0.0001
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0002)
Illiquid wealth	-0.0000	-0.0000	-0.0000	0.0000	-0.0001
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0001)
Other wealth	0.0001	-0.0003*	0.0001	-0.0000	0.0001
	(0.0002)	(0.0001)	(0.0002)	(0.0003)	(0.0003)
Debt	0.0001	-0.0000	0.0001	0.0002*	-0.0001
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0003)
log(income)	-0.0972***	-0.0826***	-0.1130***	-0.0120	-0.0035
	(0.0293)	(0.0261)	(0.0313)	(0.0364)	(0.0856)
HH Controls	Yes	Yes	Yes	Yes	Yes
Observations	1900	1900	1900	1900	567
R^2	0.0661	0.0603	0.0591	0.0237	0.0703

Note: Bundesbank-Online-panel-Households, November 2021 wave. The units of the wealth and debt variables are €1000s. The household controls are age (in years up to a top bin of ≥ 80, coded as 80), age^2, gender, region (north/south/east/west), education, occupation category, and employment status (all categorical, for details see the full questionnaire at https://www.bundesbank.de/en/bundesbank /research/survey-on-consumer-expectations/questionnaires-850746). All controls except age and age^2 are treated as categorical. Robust standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Result 5 *Higher income is associated with less uncertainty about current and future inflation, and less perceived noise in the inflation process, but is not associated with differences in perceived inflation persistence or Kalman gains.*

As documented in other contexts (e.g. Ben-David et al., 2018), higher-income households are less uncertain about future inflation: a 10% rise in household income is associated with a 1% fall in $SD_i(\pi_{t+1})$. There is however no evidence that this comes from high-income households acquiring more precise information, as they are also less uncertain about current inflation. Since the differences across current and future uncertainty are small, there is no significant relationship between income and χ_i. Rather, the bulk of the lower uncertainty for high-income households is explained by them believing the inflation process is less volatile. A 10% rise in income is associated with a 1.1% reduction in $SD_i(\varepsilon_{t+1})$. There is no significant correlation between income and $\tilde{\rho}_i$.

5 Implications for Aggregate Consumption

We now analyse how our results affect aggregate dynamics through a simple partial-equilibrium model with both unconstrained and hand-to-mouth households.

5.1 Consumption-Saving Model

Unconstrained households have an infinite horizon and no borrowing constraint. They choose consumption $\hat{c}_{i,t}$ to maximize the expected discounted sum of CRRA utility over consumption, and invest any unspent exogenous income $y_{i,t}$ in risk-free one-period bonds with gross nominal interest rate i_t. The log-linearized consumption function is:[15]

$$\hat{c}_{i,t} = \sum_{h=0}^{\infty} \beta^h \left((1-\beta)\tilde{E}_{i,t} y_{i,t+h} - \beta\gamma^{-1}\tilde{E}_{i,t} i_{t+h} + \beta\gamma^{-1}\tilde{E}_{i,t}\pi_{t+h+1} \right) \tag{12}$$

where β is the discount factor and γ is the coefficient of relative risk aversion. Note that this is the consumption function implied by standard textbook macroeconomic models. It is obtained by combining the log-linear Euler equation with the household's intertemporal budget constraint (see Gabaix (2020) Proposition 29).

To isolate the effect of a shock to expected inflation, we hold expected $y_{i,t+h}$ and i_{t+h} constant in all exercises (relaxed in Appendix E). Using equation 5, consumption is:[16]

$$\hat{c}_{i,t} = \frac{\beta\gamma^{-1}}{1-\beta\tilde{\rho}_i}\tilde{E}_{i,t}\pi_{t+1} \tag{13}$$

A higher $\tilde{\rho}_i$ therefore increases the responsiveness of consumption to expected inflation, as it implies larger changes in longer-horizon expectations.

Using equation 10 and aggregating across households, the aggregate consumption response to a one percentage-point inflation shock in $t=0$ is:

$$\hat{c}_0 = \beta\gamma^{-1}\left(E[\chi_i]E\left[\frac{\tilde{\rho}_i}{1-\beta\tilde{\rho}_i}\right] + Cov\left(\chi_i, \frac{\tilde{\rho}_i}{1-\beta\tilde{\rho}_i}\right)\right) \tag{14}$$

Heterogeneity in expectation formation therefore affects aggregate consumption in two ways. First, heterogeneity in $\tilde{\rho}_i$ amplifies the aggregate consumption responses to inflation, because $\tilde{\rho}_i/(1-\beta\tilde{\rho}_i)$ is convex in $\tilde{\rho}_i$. If even a few households believe that inflation is close to a unit root, they respond very strongly to current inflation, generating large ag-

[15] Note that by studying a log-linearized model, we abstract away from any direct effects of uncertainty on consumption. The uncertainties measured in the survey are still important, however, because they determine the responsiveness of expectations to shocks through the Kalman gain χ_i (equation 6).

[16] This assumes $|\beta\tilde{\rho}_i| < 1$. When calibrating to the data we drop the minority of households for whom this is not true.

gregate consumption responses. Note that heterogeneity in $\tilde{\rho}_i$ amplifies the consumption response *relative* to the response of inflation expectations. Similar aggregate impulse responses in inflation expectations may therefore correspond to very different impulse responses in consumption.

Second, any correlation between $\tilde{\rho}_i$ and χ_i will further distort the aggregate consumption response away from the representative-agent case. Intuitively, the response of aggregate consumption is amplified if the households who obtain precise information about the shock are also the ones who respond most strongly to that information. This is an example of the 'narrative heterogeneity channel' discussed in Macaulay (2022).

Constrained (hand-to-mouth) households, in contrast, do not respond to expectations. Since we abstract from indirect effects of nominal shocks through incomes, they have $\hat{c}_{i,t} = 0$.

5.2 Aggregate Shock Transmission

We now generate impulse responses of aggregate one-year ahead inflation expectations and consumption in three cases. First, we consider full information rational expectations (FIRE): all households know that $\rho = 0.21$, and observe π_t precisely. Second, we maintain homogeneity, and calibrate the model using the population averages for χ_i and $\tilde{\rho}_i$ in the survey. Finally, we allow for heterogeneity, calibrating to the observed joint distribution of χ_i and $\tilde{\rho}_i$.[17]

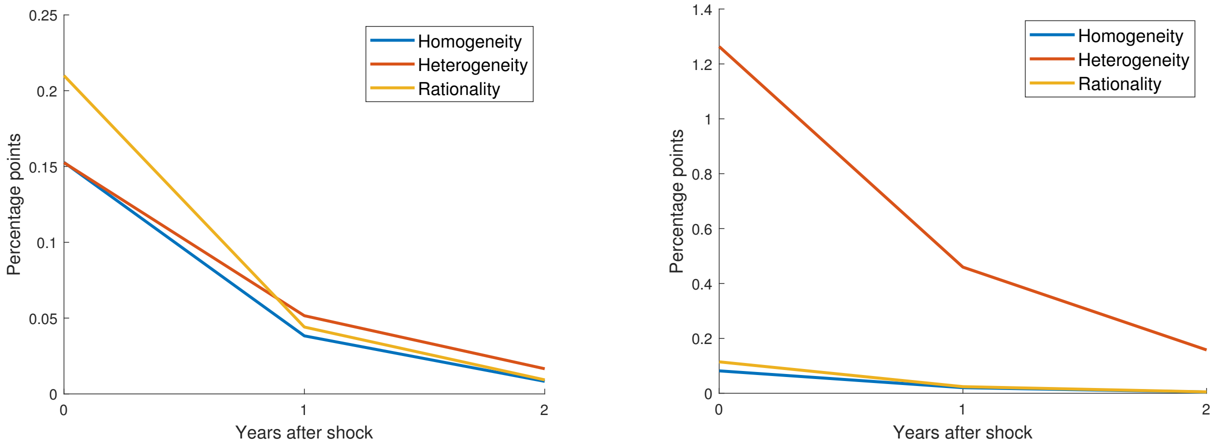
Figure 2a plots the IRFs for $\tilde{E}_t\pi_{t+1}$. Expectations respond less on impact in both cases calibrated to the survey data than with FIRE, because $E[\chi_i] < 1$ and $E[\tilde{\rho}_i] \approx \rho$. While on impact expectations in these two cases are similar, they are somewhat more persistent under heterogeneity: a year after the shock, average $\tilde{E}_{i,t}\pi_{t+1}$ is c.35% greater than under homogeneity.

Figure 2b plots the IRFs for aggregate consumption. They differ considerably between cases.

Result 6 *The model-implied consumption response under heterogeneity is* $15.5\times$ *greater on impact than under homogeneity. The persistence of the consumption response under heterogeneity* ($\frac{\hat{c}_1}{\hat{c}_0}$) *is c.45% greater than under homogeneity.*

On impact, the FIRE response is small (0.11%). It has persistence of 0.21, so the deviation from steady-state in $t = 1$ is negligible. The homogeneous case has an even smaller initial consumption response (0.08%), and marginally greater persistence.

[17] To exclude outliers, in all cases we exclude observations with $\tilde{\rho}_i \notin [0, 1]$. The excluded households are disproportionately the hand-to-mouth, who are least able to respond to expectations (Result 4).



(a) Expectations **(b)** Consumption

Figure 2: Implied IRFs of one-period ahead inflation expectations and consumption. Source: Bundesbank-Online-Panel-Households, November 2021 wave.

The heterogeneous case, however, has a vastly larger initial consumption response of 1.26%. The response is also more persistent, with a persistence of 0.36 between $t = 0$ and $t = 1$. Aggregate consumption therefore remains substantially above steady-state in the two years following the shock. In Appendix E, we show that these effects principally reflect the heterogeneity in $\tilde{\rho}_i$.

These figures demonstrate the challenges involved in inferring how expectations affect macroeconomic dynamics using only aggregate or consensus expectations data. The homogeneity and heterogeneity cases yield similar IRFs in aggregate inflation expectations, but entirely different IRFs in aggregate consumption.

Discussion. The model studied here is simple, which serves two purposes: it highlights how heterogeneity in expectation formation can have large effects even in textbook models, and it allows us to study the mechanisms behind those effects analytically. However, to achieve this we have made several simplifying assumptions. In particular, the results rely on the properties of the consumption function (equation 12), which assumes households follow an Euler equation. While we cannot test this directly in our data, Dräger and Nghiem (2021) document such behavior for German households. Hanspal et al. (2021) also find that perceived persistence is important for consumption decisions in the context of Covid-19 expectations.

The amplification result survives a number of extensions and robustness checks. Appendix A.2 demonstrates that the key channels continue to operate if households use richer subjective laws of motion. Additionally, Appendix E shows that amplification from heterogeneity remains large if the model is calibrated separately to results for each of the shock scenarios in survey Question 2, if we exclude rounded expectations, and if we allow expectations of future nominal interest rates to react to expected inflation.[18]

In general equilibrium, these effects will be amplified if the consumption increase leads to rising real incomes for hand-to-mouth households, and rising income expectations for unconstrained households. A further round of general equilibrium effects may then also occur through the Phillips curve. We leave exploration of these effects to future research.

6 Conclusion

Inflation expectations are important in many theories of the business cycle. However, the quantities measured by existing expectation surveys are consistent with many different laws of motion for expectations, with contrasting aggregate implications. To distinguish between these models, we use novel survey data to elicit (i) households' uncertainty over current inflation, and (ii) how persistent they perceive inflation to be, at the individual level.

We find that, on average, consumers are relatively confident about current inflation, and perceive little persistence in inflation. However, these averages mask considerable heterogeneity, which increases the aggregate consumption response to an inflation shock by an order of magnitude in an otherwise standard consumption-savings model. The persistence of consumption responses to shocks also increases substantially.

This effect occurs because individual consumption functions are highly non-linear in the components of expectation formation. Heterogeneity in those parameters, and correlations between them, can therefore have large effects on aggregate consumption, even for a given path of aggregate expectations. The components of expectation formation are also correlated with household wealth and income, both of which correlate with consumption behavior (Kaplan et al., 2014; Kueng, 2018). Exploring the distribution of these components for expectations of other variables, and how the distributions change over time and states of the world, could be a fruitful avenue for future research.

[18] Figures 2a and 2b can be seen as a baseline in which households believe nominal interest rates are unresponsive to inflation. This is not unreasonable, since at the time of the survey inflation in Germany had risen sharply over the preceding year, but the ECB was still a long way from raising nominal interest rates (Lagarde, 2021), and its credibility was low among German households (Coleman and Nautz, 2023).

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A Proofs

A.1 Baseline Model

Steady-State Kalman Filter

Before receiving the signal at time t, the agent's subjective distribution for π_t and the signal is:

$$\begin{pmatrix} \pi_t \\ \pi_t + q_{i,t} \end{pmatrix} \sim N\left(\begin{pmatrix} \tilde{\rho}_i \tilde{E}_{i,t-1}\pi_{t-1} \\ \tilde{\rho}_i \tilde{E}_{i,t-1}\pi_{t-1} \end{pmatrix}, \begin{pmatrix} V^f_{i,t-1} & V^f_{i,t-1} \\ V^f_{i,t-1} & V^f_{i,t-1} + \tilde{\sigma}^2_{q,i} \end{pmatrix} \right) \tag{15}$$

The conditional mean of π_t given the signal is then:

$$\tilde{E}_{i,t}\pi_t = (1 - \chi_{i,t})\tilde{\rho}_i \tilde{E}_{i,t-1}\pi_{t-1} + \chi_{i,t}(\pi_t + q_{i,t}), \text{ where } \chi_{i,t} = \frac{V^f_{i,t-1}}{V^f_{i,t-1} + \tilde{\sigma}^2_{q,i}} \tag{16}$$

The conditional variance of π_t:

$$V^p_{i,t} = V^f_{i,t-1}\left(1 - \frac{(V^f_{i,t-1})^2}{V^f_{i,t-1}(V^f_{i,t-1} + \tilde{\sigma}^2_{q,i})}\right) = \frac{V^f_{i,t-1}\tilde{\sigma}^2_{q,i}}{V^f_{i,t-1} + \tilde{\sigma}^2_{q,i}}$$

In steady state, this variance is:

$$V^p_i = \frac{V^f_i \tilde{\sigma}^2_{q,i}}{V^f_i + \tilde{\sigma}^2_{q,i}} \tag{17}$$

The steady state Kalman gain is then:

$$\chi_i = \frac{V^f_i}{V^f_i + \tilde{\sigma}^2_{q,i}} = 1 - \frac{V^p_i}{V^f_i} \tag{18}$$

Response of Inflation Expectations to Shocks

Throughout, we assume that both inflation and the agent's inflation perception start in steady state in $t = -1$. That is, $\pi_{-1} = \tilde{E}_{i,-1}\pi_{-1} = 0$. The individual inflation perception is given by equation 16. Iterating backwards to time 0, we obtain:

$$\tilde{E}_{i,t}\pi_t = \chi_i \sum_{s=0}^{t} ((1 - \chi_i)\tilde{\rho}_i)^s (\pi_{t-s} + q_{i,t-s}) \tag{19}$$

Abstracting from $q_{i,t}$, the h period ahead forecast is then:

$$\tilde{E}_{i,t}\pi_{t+h} = \chi_i \tilde{\rho}_i^h \sum_{s=0}^{t} ((1-\chi_i)\tilde{\rho}_i)^s \pi_{t-s} = \chi_i \tilde{\rho}_i^h \rho^t \sum_{s=0}^{t} ((1-\chi_i)\tilde{\rho}_i \rho^{-1})^s \varepsilon_0 \tag{20}$$

where the second equality uses that $\pi_{t-s} = \rho^{t-s}\varepsilon_0$.

Provided that $(1-\chi_i)\tilde{\rho}_i\rho^{-1} \neq 1$, then evaluating the summation and rearranging yields the result:

$$\tilde{E}_{i,t}\pi_{t+h} = \chi_i \tilde{\rho}_i^h \frac{\rho^{t+1} - ((1-\chi_i)\tilde{\rho}_i)^{t+1}}{\rho - (1-\chi_i)\tilde{\rho}_i} \varepsilon_0 \tag{21}$$

Setting $h = 1, \varepsilon_0 = 1$ yields equation 10.

Persistence of Expectations

From equation 21 we have:

$$\frac{\tilde{E}_{i,1}\pi_2}{\tilde{E}_{i,0}\pi_1} = \rho + (1-\chi_i)\tilde{\rho}_i \tag{22}$$

The persistence of the response of expectations to the shock is increasing in ρ and $\tilde{\rho}_i$, and decreasing in χ_i (assuming $\tilde{\rho}_i > 0$). If $\rho + (1-\chi_i)\tilde{\rho}_i > 1$, then the expectation rises between the period the shock hits and the period after, giving a hump-shaped response. This condition is both necessary and sufficient for a hump-shaped impulse response.

Under heterogeneity, the impact response of aggregate inflation expectations to the shock is given by equation 11. To see the role of heterogeneity in future periods, we consider the special case of $\rho = 0$, in which case:

$$\tilde{E}_t\pi_{t+1} = E[\tilde{\rho}_i^{t+1}]E[\chi_i(1-\chi_i)^t] + Cov(\tilde{\rho}_i^{t+1}, \chi_i(1-\chi_i)^t) \tag{23}$$

$\tilde{\rho}_i^{t+1}$ is strictly convex in $\tilde{\rho}_i$ for all $t \geq 1$, so heterogeneity in $\tilde{\rho}_i$ increases the time t response for $t \geq 1$. This convexity increases with t, so the persistence of the response of expectations also increases with heterogeneity in $\tilde{\rho}_i$. In addition, note that $\chi_i(1-\chi_i)^t$ is linear in χ_i for $t = 0$, concave in χ_i if $t = 1$, but becomes convex in χ_i as t becomes large. Heterogeneity in χ_i consequently decreases persistence for small t, but may increase persistence for large enough t.

To understand the role of the covariance term, consider the simple case where $\tilde{\rho}_i$ is monotonically increasing in χ_i. The covariance term is then positive for small t, but negative for large t. As such, a positive correlation between $\tilde{\rho}_i$ and χ_i tends to result in lower persistence of the response in inflation expectations, despite a larger initial response.

Formally, the persistence of aggregate expectations between $t = 0$ and $t = 1$ is:

$$\frac{\tilde{E}_1\pi_2}{\tilde{E}_0\pi_1} = \rho + \left(E[\tilde{\rho}_i] + \frac{Var(\tilde{\rho}_i)}{E[\tilde{\rho}_i]}\right)\left(1 - E[\chi_i] - \frac{Var[\chi_i]}{E(\chi_i)}\right) + \text{ covariance terms} \tag{24}$$

This highlights that heterogeneity in $\tilde{\rho}_i$ helps to generate hump-shaped IRFs in expectations, while heterogeneity in χ_i makes hump-shaped responses less likely to emerge.

Consumption responses

Consider an unconstrained agent facing an infinite-horizon consumption savings problem. As in Gabaix (2020), take income as given. The consumption function is:

$$\hat{c}_{i,t} = \sum_{h\geq 0} \beta^h ((1-\beta)\tilde{E}_{i,t}\hat{y}_{i,t+h} - \beta\gamma^{-1}\tilde{E}_{i,t}i_{t+h} + \beta\gamma^{-1}\tilde{E}_{i,t}\pi_{t+h+1}) \tag{25}$$

Since we hold expected income and nominal interest rates at steady state, we have $\tilde{E}_{i,t}\hat{y}_{i,t+h} = 0$ and $\tilde{E}_{i,t}i_{t+h} = 0$ for all t and h. The consumption function then reduces to:

$$\hat{c}_{i,t} = \beta\gamma^{-1}\sum_{h\geq 0}\beta^h \tilde{E}_{i,t}\pi_{t+h+1} = \beta\gamma^{-1}\frac{1}{1-\beta\tilde{\rho}_i}\tilde{E}_{i,t}\pi_{t+1} \tag{26}$$

To proceed, substitute in for the one period ahead expectation in time t using equation 21:

$$\hat{c}_{i,t} = \beta\gamma^{-1}\frac{1}{1-\beta\tilde{\rho}_i}\tilde{\rho}_i\chi_i\frac{\rho^{t+1}-(1-\chi_i)^{t+1}\tilde{\rho}_i^{t+1}}{\rho-(1-\chi_i)\tilde{\rho}_i} + d_{i,t} \tag{27}$$

Here, $d_{i,t}$ is an idiosyncratic noise term, which is a linear function of $q_{i,t}$, $q_{i,t-1}$, ...,$q_{i,0}$, and so has mean zero. Averaging across agents, one obtains:

$$\hat{c}_t = \beta\gamma^{-1}E\left[\frac{1}{1-\beta\tilde{\rho}_i}\tilde{\rho}_i\chi_i\frac{\rho^{t+1}-(1-\chi_i)^{t+1}\tilde{\rho}_i^{t+1}}{\rho-(1-\chi_i)\tilde{\rho}_i}\right] \tag{28}$$

Which in $t=0$ becomes:

$$\hat{c}_0 = \beta\gamma^{-1}E\left[\frac{\tilde{\rho}_i}{1-\beta\tilde{\rho}_i}\chi_i\right] \tag{29}$$

Applying the definition of a covariance then leads to equation 14.

A.2 Extended Subjective Laws of Motion

Suppose households hold a subjective law of motion for inflation of the form:

$$\pi_{t+1} = \tilde{\rho}_i\pi_t + \xi_t + \varepsilon_{t+1} \tag{30}$$

$$\varepsilon_{t+1} \sim N(0, \tilde{\sigma}^2_{\varepsilon i}) \tag{31}$$

where ξ_t is (perceived to be) a normally distributed random variable. Inflation and this new term are therefore jointly normal:

$$\begin{pmatrix} \pi_t \\ \xi_t \\ \varepsilon_{t+1} \end{pmatrix} \sim N \left(\begin{pmatrix} \tilde{\mu}_{\pi i} \\ \tilde{\mu}_{\xi i} \\ 0 \end{pmatrix}, \begin{pmatrix} \tilde{\sigma}^2_{\pi i} & \tilde{r}_i & 0 \\ \tilde{r}_i & \tilde{\sigma}^2_{\xi i} & \tilde{r}_{\varepsilon i} \\ 0 & \tilde{r}_{\varepsilon i} & \tilde{\sigma}^2_{\varepsilon i} \end{pmatrix} \right) \tag{32}$$

where $\tilde{r}_i$ is household i's perception of the covariance between π_t and ξ_t, and $\tilde{r}_{\varepsilon i}$ is the perceived covariance between ξ_t and ε_{t+1}. $\tilde{\mu}_{\pi i}, \tilde{\mu}_{\xi i}$ are the perceived means of each variable. From equation 30, $\tilde{\mu}_{\xi i} = (1 - \tilde{\rho}_i)\tilde{\mu}_{\pi i}$.

ξ_t could include, for example, longer lags of inflation, long-run inflation expectations, or realizations of other variables such as output or interest rates. The key restriction is that ξ_t is included additively in the law of motion, and is normally distributed. This is necessary to ensure that the Kalman filter remains the appropriate way for households to interpret signals on inflation.

With this extended law of motion, Bayesian updating implies that household i uses $s_{i,t}$ to form perceptions according to:[19]

$$\tilde{E}_{i,t}\pi_t = (1 - \chi_i)(\tilde{\rho}_i \tilde{E}_{i,t-1}\pi_{t-1} + \tilde{E}_{i,t-1}\xi_t) + \chi_i(\pi_t + q_{it}) \tag{33}$$

$$\tilde{E}_{i,t}\xi_t = \tilde{E}_{i,t-1}\xi_t + \frac{\tilde{r}_i}{V_i^f}\chi_i(\pi_t + q_{i,t} - \tilde{E}_{i,t-1}\pi_t) \tag{34}$$

where:

$$\chi_i = \frac{V_i^f}{V_i^f + \tilde{\sigma}^2_{q,i}} \tag{35}$$

The Kalman gain, reflecting the sensitivity of inflation perceptions to realized inflation, therefore has exactly the same form as in our baseline model (equation 16). The only difference is that V_i^f now also includes uncertainty from ξ_t, and any related covariance terms. Since uncertainty about π_t is still Gaussian, V_i^p is calculated exactly as in equation 17. We can therefore measure the Kalman gain using equation 6, which implies the perceived signal noise is given by equation 8.

The inclusion of ξ_t in the subjective law of motion does however affect other aspects of perceptions. The prior belief over inflation in equation 33 now includes the prior perception of ξ_t, and if $\tilde{r}_i \neq 0$ household i uses the inflation signal to update beliefs about

[19] As in the baseline model, we restrict attention to the steady state Kalman filter here.

ξ_t as well as π_t. In addition, the variance calculated in equation 7 will now measure:

$$V_i^f - \tilde{\rho}_i^2 V_i^p = \tilde{\sigma}_{\varepsilon i}^2 + \tilde{\sigma}_{\xi i}^2 + 2\tilde{\rho}_i \tilde{r}_i + 2\tilde{r}_{\varepsilon i} \tag{36}$$

That is, this measurement combines uncertainty from inflation shocks (as in the baseline model) with uncertainty arising from the extra term in the law of motion, including from its covariance with either inflation or shocks.

We now turn to the response of expectations to inflation shocks in this extended model. As in Section 2, suppose that in period -1, household i's expectations of all variables are at steady state ($E_{i,-1}\pi_{-1} = \tilde{\mu}_{\pi i}, E_{i,-1}\xi_{-1} = \tilde{\mu}_{\xi i}$). In that case, abstracting from signal noise as in equation 10, we have:

$$\tilde{E}_{i,0}\pi_1 = \left(\tilde{\rho}_i(1-\chi_i) - \frac{\tilde{r}_i}{V_i^f}\chi_i\right)\tilde{\mu}_{\pi i} + \tilde{\mu}_{\xi i} + \left(\tilde{\rho}_i\chi_i + \frac{\tilde{r}_i}{V_i^f}\chi_i\right)\pi_0 \tag{37}$$

The change in the aggregate inflation expectation when π_0 rises by one unit is then:[20]

$$\Delta\tilde{E}_0\pi_1 = E[\tilde{\rho}_i]E[\chi_i] + Cov(\tilde{\rho}_i, \chi_i) + E[\tilde{r}_i/V_i^f]E[\chi_i] + Cov(\tilde{r}_i/V_i^f, \chi_i) \tag{38}$$

The first two terms are exactly as in equation 11, so our key channels are still present. However, they are now supplemented with two additional channels, reflecting the same mechanisms for expectations of ξ_t whenever $\tilde{r}_i \neq 0$ for at least some households.

Specific examples of extended subjective laws of motion

Suppose ξ_t reflects the long-run mean of inflation. For simplicity, assume that while long-run expectations may be heterogeneous across households (heterogeneous $\tilde{\mu}_{\xi i}$), they are constant at the household level ($\tilde{\sigma}_{\xi i}^2 = 0$). In that case, $\tilde{r}_i = 0$ for all households, as ξ_t has no variance. From equations 37, expectations are therefore exactly the same as in the baseline case, plus an individual-specific constant term. This constant does not affect the response of expectations to shocks (equation 38), so all results on shock transmission are identical to those in Sections 2 and 5. The same will apply if ξ_t reflects the forward information studied in Goldstein and Gorodnichenko (2022).

However in other cases the new terms in equation 38 will be non-zero. For example, households who believe in a wage-price spiral may believe that current incomes affect future inflation ($\xi_t = \kappa_i y_t$), and that current income comoves positively with current inflation ($\tilde{r}_i \geq 0$). Through equation 38, that implies that the channels studied in Section 2 understate the response of aggregate inflation expectations to shocks, especially if the

[20] We use this rather than the raw aggregate inflation expectation because steady state priors are now no longer necessarily zero or equal across households in this extended model. If $\tilde{\mu}_{\pi i} = \tilde{\mu}_{\xi i} = 0$ for all i then this exercise is identical to that in equation 11.

households who believe in a very strong spiral (high $\tilde{r}_i$) are systematically those obtaining more precise signals about inflation (high χ_i).

B Survey details

B.1 Summary statistics

Table 4 shows summary statistics for the key variables used in our analysis, and several other household characteristics. The construction of $SD_i(\pi_t), SD_i(\pi_{t+1}), SD_i(\varepsilon_{t+1}), \chi_i, \tilde{\rho}_i$ is described in Appendix C.1.

Income and wealth variables are reported in bins. We take the mid-point of each bin. We code the lowest bin for income as if the lower bound is zero, and again take the midpoint (all wealth variables have a separate bin for zero). The top bin is coded as if it had the same width as the second-highest bin. Liquid wealth is (bank deposits + securities). Illiquid wealth is (property + firm ownership). Debt is (secured + unsecured debt). A respondent is classified as hand-to-mouth if their liquid wealth is $<$ €1250.

Table 4: Summary statistics for expectations (point estimates and components), respondent characteristics, and income/wealth.

	Mean	Std Dev.	Min	Max
Panel A: Expectations				
$\tilde{E}_{i,t}\pi_t$	4.10	2.61	0	30
$\tilde{E}_{i,t}\pi_{t+1}$	4.90	4.99	-3	60
$SD_i(\pi_t)$	1.75	4.70	0.41	40.41
$SD_i(\pi_{t+1})$	1.72	1.35	0.30	8.80
$SD_i(\varepsilon_{t+1})$	1.87	1.52	0.04	12.12
χ_i	0.80	0.23	0	1
$\tilde{\rho}_i$	0.29	0.84	-5	5
Panel B: Demographics				
Age	56.87	14.66	16	80
Female	0.37	0.48	0	1
Higher Education	0.39	0.49	0	1
Is Working	0.55	0.50	0	1
Panel C: Income and Wealth				
Income	3.95	1.97	0.25	11
Liquid Wealth	90.49	154.90	0	1250
Illiquid Wealth	315.38	383.64	0	2375
Other Wealth	12.41	48.75	0	625
Debt	47.89	109.34	0	955
Owns Securities	0.62	0.48	0	1
Hand-to-mouth	0.14	0.34	0	1

Note: Bundesbank-Online-Panel-Households, November 2021 wave. For cases where χ_i is set-identified, respondents are excluded if the parameters are estimated very imprecisely (range>0.2). For all remaining set-identified parameters, the mid-point of the range is used. Observations of $\tilde{E}_{i,t}\pi_t, \tilde{E}_{i,t}\pi_{t+1}, SD_i(\pi_{t+1})$, and $SD_i(\pi_t)$ below the 1st or above the 99th percentile of that variable's distribution are also excluded as outliers, as are observations of $\tilde{\rho}_i$ outside $[-5, 5]$ (c.1% of observations). All income and wealth variables are in €1000s, and income refers to monthly net income of the household. Higher Education is an indicator for if the respondent has a bachelor's degree or higher, not including vocational training.

B.2 Survey questions (English and German)

Table 5 contains the existing questions in the Bundesbank survey eliciting point estimates of current and future inflation, and the density forecast of future inflation.

B.3 Novel questions for November 2021 (in German)

Table 6 contains the German text of our novel survey questions.

Table 5: Existing questions in the BOP-HH survey

Label	Text
Inflation Development	
Question	What do you think the rate of inflation or deflation in Germany was over the past twelve months?
Note	If you assume there was deflation, please enter a negative value. Values may have one decimal place.
Question	Wass denken Sie, wie hoch war die Inflationsrate oder Deflationsrate in den letzten zwölf Monaten in Deutschland?
Note	Im Falle einer angenommenen Deflationsrate tippen Sie bitte einen negativen Wert ein. Die Eingabe maximal einer Nachkommastelle ist möglich. Bitte geben Sie einen Wert hier ein.
Input Field	Percent
Inflation Expectations Qualitative	
Question	Do you think inflation or deflation is more likely over the next twelve months?
Note	Inflation is the percentage increase in the general price level. It is mostly measured using the consumer price index. A decrease in the price level is generally described as “deflation”.
Question	Was denken Sie, ist in den kommenden zwölf Monaten eher mit einer Inflation oder einer Deflation zu rechnen?
Note	Inflation ist der prozentuale Anstieg des allgemeinen Preisniveaus. Sie wird meist über den Verbraucher-preisindex gemessen. Ein Rückgang des Preisniveaus wird gemeinhin als “Deflation” bezeichnet.
Input Field	Select one answer
Inflation Expectations Quantitative	
Question	What do you think the rate of inflation/deflation will roughly be over the next twelve months? (select based on answer to *Inflation Expectations Qualitative*)
Note	Inflation is the percentage increase in the general price level. It is mostly measured using the consumer price index. A decrease in the price level is generally described as “deflation”.
Question	Was denken Sie, wie hoch wird die Inflationsrate/Deflationsrate in den kommenden zwölf Monaten in etwa sein? (select based on answer to *Inflation Expectations Qualitative*)

Note — Inflation ist der prozentuale Anstieg des allgemeinen Preisniveaus. Sie wird meist über den Verbraucherpreisindex gemessen. Ein Rückgang des Preisniveaus wird gemeinhin als “Deflation” bezeichnet. Bitte tippen Sie einen Wert in das Zahlenfeld ein (eine Nachkommastelle möglich).

- - -

Input Field — Percent

Inflation Expectations Probabilistic

Question — In your opinion, how likely is it that the rate of inflation will change as follows over the next twelve months?

Note — The aim of this question is to determine how likely you think it is that something specific will happen in the future. You can rate the likelihood on a scale from 0 to 100, with 0 meaning that an event is completely unlikely and 100 meaning that you are absolutely certain it will happen. Use values between the two extremes to moderate the strength of your opinion. Please note that your answers to the categories have to add up to 100.

Input Field —
The rate of deflation (opposite of inflation) will be 12% or higher. __
The rate of deflation (opposite of inflation) will be between 8% and less than 12%. __
The rate of deflation (opposite of inflation) will be between 4% and less than 8%. __
The rate of deflation (opposite of inflation) will be between 2% and less than 4%. __
The rate of deflation (opposite of inflation) will be between 0% and less than 2%. __
The rate of inflation will be between 0% and less than 2%. __
The rate of inflation will be between 2% and less than 4%. __
The rate of inflation will be between 4% and less than 8%. __
The rate of inflation will be between 8% and less than 12%. __
The rate of inflation will be 12% or higher. __

- - -

Question — Für wie wahrscheinlich halten Sie es, dass sich die Inflationsrate in den kommenden zwölf Monaten wie folgt entwickelt?

Note — Bei dieser Frage geht es darum, wie Sie die Wahrscheinlichkeit einschätzen, dass ein bestimmter Sachverhalt in der Zukunft eintritt. Ihre Antworten können in einer Spanne zwischen 0 und 100 liegen, wobei 0 absolut unwahrscheinlich bedeutet und 100 absolut sicher. Mit Werten dazwischen können Sie Ihre Einschätzung abstufen. Bitte beachten Sie, dass sich die Angaben über alle Kategorien auf 100 summieren müssen.

Input Field —
die Deflationsrate (Gegenteil von Inflation) wird 12% oder höher sein. __
die Deflationsrate (Gegenteil von Inflation) wird zwischen 8% und 12% liegen. __
die Deflationsrate (Gegenteil von Inflation) wird zwischen 4% und 8% liegen. __
die Deflationsrate (Gegenteil von Inflation) wird zwischen 2% und 4% liegen. __
die Deflationsrate (Gegenteil von Inflation) wird zwischen 0% und 2% liegen. __
die Inflationsrate wird zwischen 0% und 2% liegen. __
die Inflationsrate wird zwischen 2% und 4% liegen. __
die Inflationsrate wird zwischen 4% und 8% liegen. __
die Inflationsrate wird zwischen 8% und 12% liegen. __
die Inflationsrate wird 12% oder höher sein. __

Table 6: Novel questions added to the BOP-HH survey in November 2021

Question	Text
1	Nun möchten wir wissen, wie sicher Sie sich über Ihre Angabe zur Inflationsrate oder Deflationsrate in den letzten 12 Monaten sind ([Value of point estimate])%. Wie wahrscheinlich ist es Ihrer Meinung nach, dass die Inflationsrate in den letzten zwölf Monaten zwischen [Low inflation level]% und [High inflation level]% lag?
Hinweis	Bei dieser Frage geht es darum, wie Sie die Wahrscheinlichkeit einschätzen, dass die von Ihnen angegebene Inflationsrate oder Deflationsrate in den letzten 12 Monaten tatsächlich ungefähr diesen Wert angenommen hat. Ihre Antworten können zwischen 0 und 100 liegen, wobei 100 bedeutet, dass Sie absolut sicher sind. Kleinere Zahlen bedeuten, dass Sie sich weniger sicher sind.
Input Field	Prozent
Respondents randomly shown one of three scenarios before Question 2	
Group A	Stellen Sie sich die folgende hypothetische Situation vor: Aufgrund eines unerwarteten wirtschaftlichen Ereignisses hat sich die Inflationsrate im vergangenen Jahr um einen Prozentpunkt erhöht.
Group B	Stellen Sie sich die folgende hypothetische Situation vor: Aufgrund von uner-warteten Problemen mit der lokalen Produktionstechnologie im Nahen Osten ist der Rohölpreis im vergangenen Jahr gestiegen, was zu einem Anstieg der Inflationsrate um einen Prozentpunkt geführt hat.
Group C	Stellen Sie sich die folgende hypothetische Situation vor: Aufgrund gestiegener Verteidigungsausgaben sind die Staatsausgaben im vergangenen Jahr unerwartet stärker als üblich gestiegen, was zu einem Anstieg der Inflationsrate um einen Prozentpunkt geführt hat. Die Änderung ist vorübergehend und tritt ein, obwohl sich die Einschätzung der Regierung zur nationalen Sicherheit oder den wirtschaftlichen Bedingungen nicht geändert hat. Darüber hinaus ändern sich die Steuern nicht als Reaktion auf das Ausgabenprogramm.
2	Würden Sie in dieser Situation Ihre im vorderen Teil des Fragebogens genannten Inflationserwartungen für die nächsten 12 Monate anpassen? Wenn ja, inwiefern?
Input Field	1) Ja, von [Value of point estimate]Prozent auf __Prozent 2) Nein

C Variable construction

C.1 Main variables

To obtain V_i^p, we fit a symmetric triangular distribution to household i's answers:

$$V_i^p = \begin{cases} \frac{1}{6}\left(1 - \sqrt{1 - \frac{x_{1i}}{100}}\right)^{-2} & \text{if } \tilde{E}_i(\pi_t) \in (-5, 5) \\ \frac{2}{3}\left(1 - \sqrt{1 - \frac{x_{1i}}{100}}\right)^{-2} & \text{if } \tilde{E}_i(\pi_t) \notin (-5, 5) \end{cases} \tag{39}$$

where x_{1i} is respondent i's response to Question 1. Note that for households who report $x_i = 0$, this method provides an upper bound on their $Var_i(\pi_t)$.

To obtain $\tilde{\rho}_i$, we set $\tilde{\rho}_i = 0$ for households who select 'No' in answer to Question 2. For all others, we set:

$$\tilde{\rho}_i = x_{2i} - \tilde{E}_i(\pi_{t+1}) \tag{40}$$

where x_{2i} is respondent i's response to Question 2.

We then calculate V_i^f. For agents who are certain future inflation will lie within one specific bin, we calculate an upper bound on the variance using the symmetric triangular distribution, just as for the perception. The lower bound on V_i^f is given by zero.

For the remaining agents, we calculate V_i^f by taking the midpoints of each of the bins in the probability distribution. Denote these midpoints as z_j for the bins $j = 1, ..., n$. Denote the probability assigned to each bin as p_j. We then calculate the mean:

$$\bar{z}_i = \sum_{j=1}^{n} p_{i,j} z_j \tag{41}$$

The variance is then:

$$V_i^f = \sum_{j=1}^{n} p_{i,j}(z_j - \bar{z}_i) \tag{42}$$

This amounts to fitting a step-wise CDF to the household's answers to the density forecast question.

The calculation of the Kalman gain is complicated by the fact that that for some respondents we have ranges of possible V_i^p or V_i^f, in which case we can only find ranges for χ_i and the other key parameters. We now describe how we calculate these parameters for each such case.

Case (i): V_i^p and V_i^f both point-identified

Calculate χ_i using:

$$\chi_i = 1 - \frac{V_i^p}{V_f^i} \tag{43}$$

Back out $\tilde{\sigma}^2_{\varepsilon,i}$ and $\tilde{\sigma}^2_{q,i}$ using:

$$\tilde{\sigma}^2_{\varepsilon,i} = V_i^f - \tilde{\rho}_i^2 V_i^p \tag{44}$$

$$\tilde{\sigma}^2_{q,i} = \frac{V_i^f V_i^p}{V_i^f - V_i^p} \tag{45}$$

Datapoints are inconsistent with Kalman filtering (and so are dropped) if $\chi_i < 0$ or $\tilde{\sigma}^2_{\varepsilon,i} < 0$.

Case (ii): V_i^f point-identified, V_i^p set-identified

This occurs if the respondent is certain that π_t lies within the specified interval, but places strictly positive probability in multiple intervals in the expectation question. V_i^p is then bounded below by zero, and the upper bound is calculated using the symmetric triangular distribution as above.

Denote the upper bound on V_i^p by a, so that $V_i^p \in [0, a]$. Under steady state Kalman filtering, it must be that $V_i^p \le V_i^f$ and $V_i^p \le \tilde{\rho}_i^{-2} V_i^f$. The latter is more restrictive if $|\tilde{\rho}_i| > 1$. This may shrink the upper bound on V_i^p, and hence raise the lower bound on the Kalman filter. As such, $V_i^p \in [0, \tilde{a}]$, where $\tilde{a}$ is given by:

$$\tilde{a} = \min(V_i^f, \tilde{\rho}_i^{-2} V_i^f, a) \tag{46}$$

Then we have the following ranges for the key parameters:

$$\chi_i \in \left[1 - \frac{\tilde{a}}{V_i^f}, 1\right], \ \tilde{\sigma}_{q,i}^2 \in \left[0, \frac{V_i^f \tilde{a}}{V_i^f - \tilde{a}}\right], \ \tilde{\sigma}_{\varepsilon,i}^2 \in \left[V_i^f - \tilde{\rho}_i^2 \tilde{a}, V_i^f\right] \tag{47}$$

Case (iii): V_i^f set-identified, V_i^p point-identified

In this case, the consumer is not certain that current inflation lies within the specified interval, but is certain that future inflation lies within one specific interval. As such, V_i^p is known, but $V_i^f \in [0, b]$, where b is given by the symmetric triangular distribution.

Under steady state Kalman filtering, it must be the case that $V_i^f \ge V_i^p$ and $V_i^f \ge \tilde{\rho}_i^2 V_i^p$. Hence $V_i^f \in [\tilde{b}, b]$, where:

$$\tilde{b} = \max(V_i^p, \tilde{\rho}_i^2 V_i^p) \tag{48}$$

Note that if $\tilde{b} > b$, then the observations must be dropped as they are inconsistent with steady state Kalman filtering. Using the equation for the Kalman gain, we then have:

$$\chi_i \in \left[1 - \frac{V_i^p}{\tilde{b}}, 1 - \frac{V_i^p}{b}\right] \tag{49}$$

The variance of the signal then lies in the interval:

$$\tilde{\sigma}_{q,i}^2 \in \left[\frac{b V_i^p}{b - V_i^p}, \frac{\tilde{b} V_i^p}{\tilde{b} - V_i^p}\right] \tag{50}$$

Note that if $\tilde{b} = V_i^p$, then the upper end of this interval is infinite, implying the signal may be infinitely noisy (i.e. contains no information).

Finally, the perceived variance of the shock lies in the range:

$$\tilde{\sigma}_{\varepsilon,i}^2 \in [\tilde{b} - \tilde{\rho}_i^2 V_i^p, b - \tilde{\rho}_i^2 V_i^p] \tag{51}$$

Case (iv): V_i^f and V_i^p both set-identified

In this case, the consumer is certain that current inflation lies within the specified interval, and certain that future inflation will lie within one specific interval. Hence, we have $V_i^p \in [0,a]$ and $V_i^f \in [0,b]$. If $|\tilde{\rho}_i| \leq 1$, then χ_i is unrestricted within the interval $[0,1]$. If $|\tilde{\rho}_i| > 1$, then χ_i is bounded below as described above. Hence $\chi_i \in [0,1]$ if $|\tilde{\rho}_i| \leq 1$, and $\chi_i \in [1-\tilde{\rho}_i^{-2}, 1]$ if $|\tilde{\rho}_i| > 1$.

We then know that:

$$\tilde{\sigma}_{q,i}^2 = \frac{V_i^f V_i^p}{V_i^f - V_i^p} \tag{52}$$

If $|\tilde{\rho}_i| < 1$, this can take any value. It could be infinite large if $V_i^p = V_i^f$, and could be zero if $V_i^p = 0$ but $V_i^f > 0$. If $|\tilde{\rho}_i| > 1$, then $V_i^f \geq \tilde{\rho}_i^2 V_i^p$. In that case, $\tilde{\sigma}_{q,i}^2$ could still be zero, but the maximum value it can now take is:

$$\tilde{\sigma}_{q,i}^2 = \frac{V_i^f V_i^p}{V_i^f - V_i^p} \leq \frac{V_i^f V_i^p}{\tilde{\rho}_i^2 V_i^p - V_i^p} \tag{53}$$

$$= \frac{V_i^f}{\tilde{\rho}_i^2 - 1} \leq \frac{b}{\tilde{\rho}_i^2 - 1} \tag{54}$$

To summarize, then, $\tilde{\sigma}_{q,i}^2 \in [0,\infty)$ if $|\tilde{\rho}_i| \leq 1$, and $\tilde{\sigma}_{q,i}^2 \in [0, \frac{b}{\tilde{\rho}_i^2 - 1}]$ if $|\tilde{\rho}_i| > 1$.

Turning to $\tilde{\sigma}_{\varepsilon,i}^2$, this could always be zero in this case. The maximum it could be is b if $V_i^f = b$ and $V_i^p = 0$. Hence $\tilde{\sigma}_{\varepsilon,i}^2 \in [0,b]$.

C.2 Alternative measurement for V_i^f

We consider two alternative measures for V_i^f. Like the measurement of V_i^p, both make use of just two pieces of information per respondent: their point estimate for inflation in the following year, and the probability that inflation will be within a particular range around that point estimate. We use these two pieces of information to fit a symmetric triangular distribution to beliefs about future inflation, and infer the variance from that.

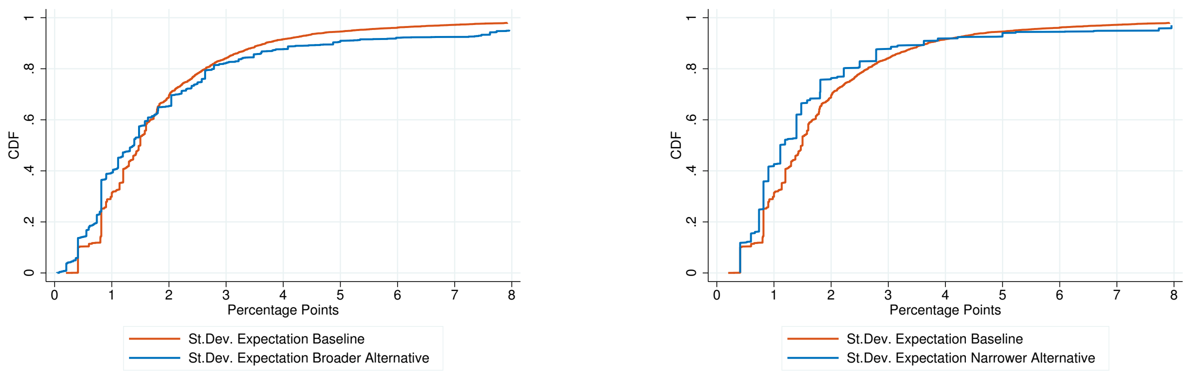
For both of these measures, we take the point estimate for future inflation from the existing question in the survey (see Table 5). For the first (broader) measure, we then consider the density forecast question, and focus just on the inflation rate bin containing the respondent's point estimate. The probability assigned to this inflation range gives us the second piece of information. That is, we observe:

1. $\tilde{E}_{i,t}\pi_{t+1}$
2. $\Pr(lb < \pi_{t+1} \leq ub)$

for lb, ub defined by the edges of the relevant bin in the density forecast. We then fit the symmetric triangular distribution as described in Appendix C.1. If the point estimate is on the boundary between two bins in the density forecast, we combine the bins to form one wider inflation range, and take the sum of the probabilities given. This disregards some information contained within the future inflation density forecasts, and indeed requires dropping a small number of observations where the point estimate is completely inconsistent with the density forecast (i.e. the density forecast assigns 0 probability to the bin containing the point estimate). The sample size therefore shrinks somewhat, to c.93% of the original sample size. It is however much closer to the measurement of V_i^p: the only differences are that the respondent has been simultaneously asked about the probabilities of inflation being in several ranges rather than just one, and that the bin we use is not necessarily symmetric about their point estimate.

In the second (narrower) alternative measure for V_i^f, we go further and remove the second of these points of difference. That is, we restrict the sample to only respondents whose point estimate is at the mid-point of one of the bins in the density forecast question, then apply the same method described above. This substantially reduces the number of observations, but does leave us with a measure of V_i^f computed in the same way as V_i^p. The only assumption required to make them exactly comparable is an independence of irrelevant alternatives: the fact that respondents are also asked about the probability of inflation being in other ranges far away from their point estimate does not affect their answer for the range around their point estimate.

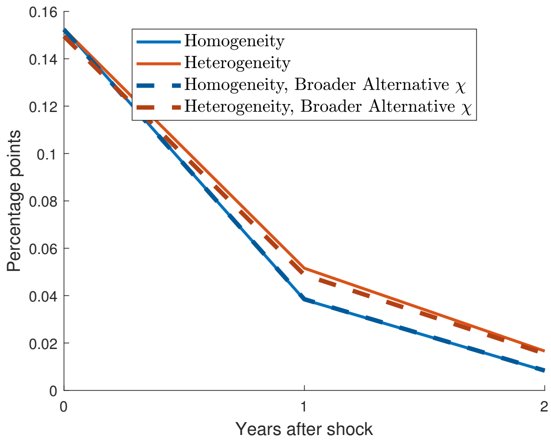
Figure 3 shows the distributions of V_i^f computed using our baseline measure using all of the information in density forecasts, and our two alternative triangular measures. They are all extremely similar. Moreover, the ranking of individuals within these distributions is strongly correlated. The Spearman's rank correlations of the first and second alternative measures with the baseline measure across individuals is 0.76 and 0.84 respectively.



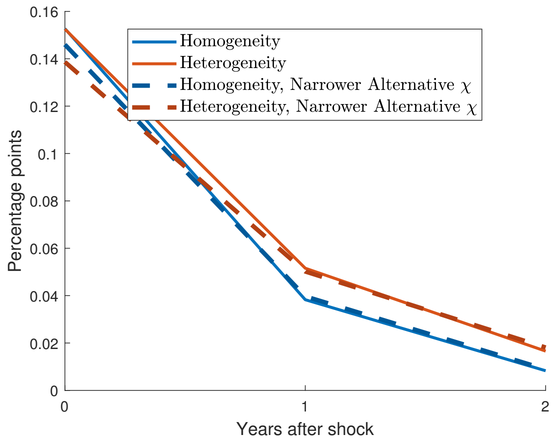
(a) Broader Alternative Measure **(b)** Narrower Alternative Measure

Figure 3: CDF of $St.Dev_i(\pi_{t+1})$ under the narrower and broader alternative calculation measures for V_i^f. Source: Bundesbank-Online-Panel-Households, November 2021 wave.

Unsurprisingly, the key results are therefore robust to these alternative variance measures. The mean Kalman gains in the two cases are 0.79 and 0.72, similar to to the 0.80 we find using our baseline measure. The different ways of calculating the Kalman gain correlate strongly across individuals, giving a Spearman's rank correlation of our baseline measure of χ_i with the first (broader) alternative of 0.69, and with the second (narrower) measure of 0.75. The impulse responses to an inflation shock in the model calibrated using the alternative measures are extremely close to those using the baseline measure (figures 4 and 5).

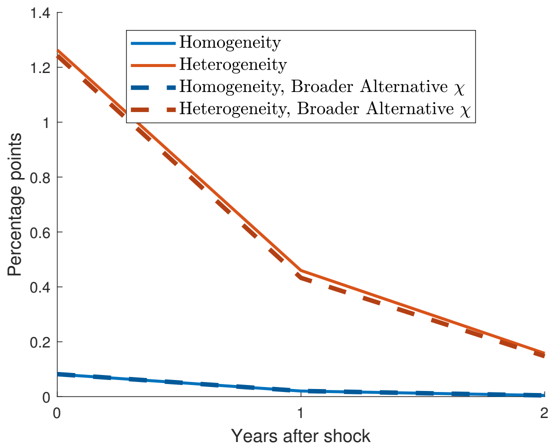


(a) Broader Alternative Measure

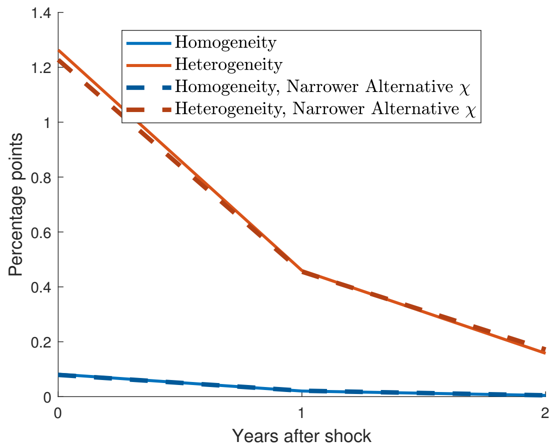


(b) Narrower Alternative Measure

Figure 4: IRF of $\tilde{E}_t\pi_{t+1}$ under the narrower and broader alternative calculation measures for V_i^f. Source: Bundesbank-Online-Panel-Households, November 2021 wave.



(a) Broader Alternative Measure



(b) Narrower Alternative Measure

Figure 5: IRF of $\hat{c}_t$ under the narrower and broader alternative calculation measures for V_i^f. Source: Bundesbank-Online-Panel-Households, November 2021 wave.

D Additional empirical results

D.1 Google trends

Figure 6 shows the time series of Google search intensity for the term "inflation" in Germany from January 2019 to July 2023. Search intensity is normalized so that the peak intensity has a value of 100. Search intensity was 83% higher in November 2021 than in January 2021, but was still less than half of its peak in September 2022.

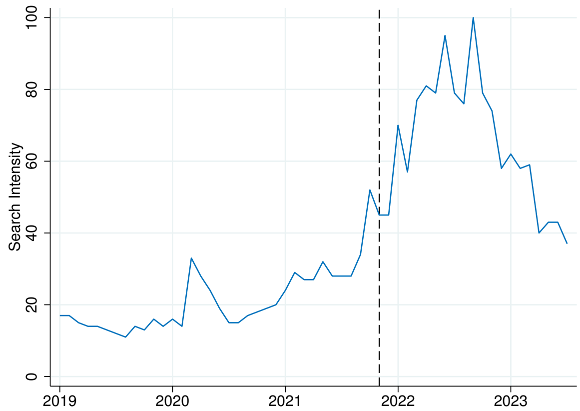
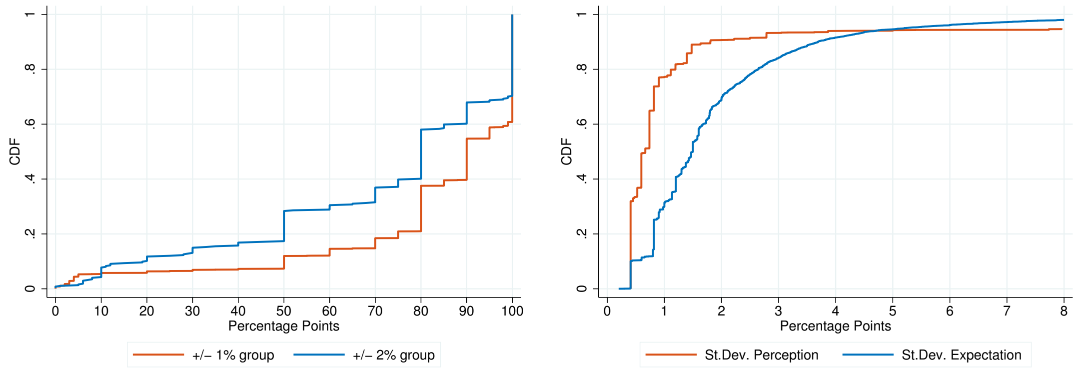


Figure 6: Internet search intensity in Germany for "inflation", January 2019 - July 2023. Search volumes are normalized so September 2022=100. The dashed line denotes November 2021, when the survey wave containing the novel questions in Table 1 was run. Source: Google Trends.

D.2 Additional parameter distributions

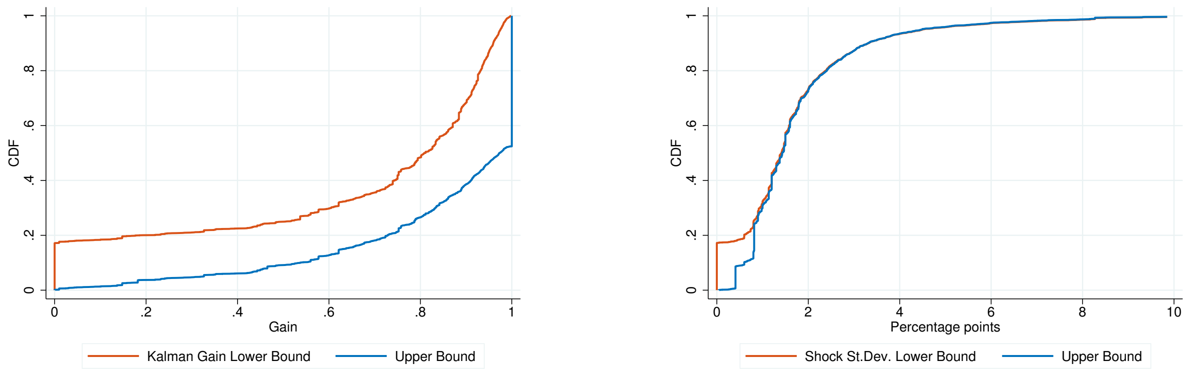
Figure 7a shows the CDF of the raw responses to question 1: respondents' assessment of the probability that current inflation lies within the specified range around their point estimate. We split the data between those with $\tilde{E}_{i,t}\pi_t \in (-5, 5)$, who were shown a $\pm 1\%$ interval, and those with $\tilde{E}_{i,t}\pi_t$ outside of this range, who were shown a $\pm 2\%$ interval. In both distributions, the majority believe there is at least an 80% chance that inflation lies within that range. Note that the $\pm 1\%$ group are more confident, despite seeing a smaller range, consistent with the notion that those who perceive lower rates of inflation or deflation are more certain in their perceptions. Figure 7b plots the CDFs of V_i^p and V_i^f. In cases where these are only set-identified, this plots the upper bound from fitting a symmetric triangular distribution. The lower bound in all such cases is 0. On average households are less uncertain about current inflation than about future inflation.

For respondents where we can only identify ranges for V_i^p and V_i^f, we can similarly only identify bounds for $\chi_i, \tilde{\sigma}^2_{q,i}, \tilde{\sigma}^2_{\varepsilon,i}$. Figure 8 shows the distributions of these parameters if we take the upper or lower bounds of the parameter ranges for those households respectively.

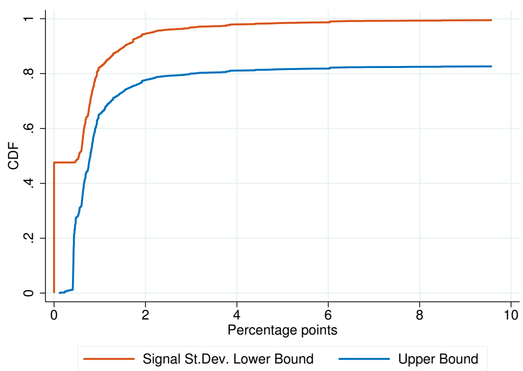


(a) CDFs of responses to question 1. **(b)** CDFs of V_i^p and V_i^f.

Figure 7: CDF of raw responses to question 1, for both the group shown a ±1% range and the group show a ±2% range, and the implied CDFs of V_i^p and V_i^f. Source: Bundesbank-Online-Panel-Households, November 2021 wave.



(a) χ_i **(b)** $\tilde{\sigma}_{\varepsilon,i}$



(c) $\tilde{\sigma}_{q,i}$

Figure 8: CDFs of upper and lower bounds for inferred parameters. Source: Bundesbank-Online-Panel-Households, November 2021 wave.

Figure 9 plots the distribution of $\tilde{\rho}_i$ when we exclude respondents whose response to the initial inflation expectations question ends in .0 or .5. Even excluding these households with the strongest tendency to round their answers, there is a large mass with $\tilde{\rho}_i = 0$, and substantial heterogeneity.

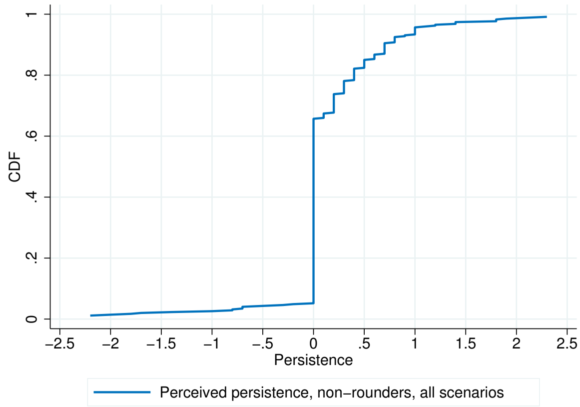


Figure 9: CDF of perceived persistence, only respondents whose response to initial inflation expectations question does not end in .0 or .5. Source: Bundesbank-Online-Panel-Households, November 2021 wave.

D.3 Relationship between components of expectation formation and point estimates

Table 7 shows the means of the elements of the expectation laws of motion, broken down by the respondent's inflation perception ($\tilde{E}_{i,t}\pi_t$). Those with an inflation perception far away from the actual value (which was approximately 5% at the time of the survey) tend to be the least certain in their perceptions. Those with the highest perceptions are also the least certain in their expectations and perceive the noise in the inflation process to be the highest. Those with perceptions that are either very high or very low also tend to have very low perceived persistence on average, and lower Kalman gains.

Table 7: Means of elements of expectation laws of motion, by inflation perception

	$SD_i(\pi_t)$	$SD_i(\pi_{t+1})$	$SD_i(\varepsilon_{t+1})$	χ_i	$\tilde{\rho}_i$
$\tilde{E}_{i,t}\pi_t < 0$	1.07	2.60	2.53	0.67	-0.13
	(0.27)	(0.52)	(0.48)	(0.13)	(0.30)
$\tilde{E}_{i,t}\pi_t \in [0,2)$	0.77	1.85	1.80	0.70	0.21
	(0.04)	(0.16)	(0.17)	(0.04)	(0.08)
$\tilde{E}_{i,t}\pi_t \in [2,4)$	0.67	1.84	1.78	0.78	0.24
	(0.01)	(0.03)	(0.03)	(0.01)	(0.02)
$\tilde{E}_{i,t}\pi_t \in [4,6)$	0.71	2.13	2.07	0.81	0.22
	(0.01)	(0.04)	(0.04)	(0.01)	(0.02)
$\tilde{E}_{i,t}\pi_t \in [6,8)$	1.44	3.35	3.22	0.74	0.22
	(0.10)	(0.19)	(0.19)	(0.03)	(0.09)
$\tilde{E}_{i,t}\pi_t \in [8,10)$	1.44	3.95	3.89	0.81	0.17
	(0.17)	(0.44)	(0.41)	(0.07)	(0.17)
$\tilde{E}_{i,t}\pi_t \geq 10$	2.11	4.83	4.73	0.72	0.04
	(0.18)	(0.29)	(0.30)	(0.04)	(0.07)

Note: Bundesbank-Online-panel-Households, November 2021 wave. Standard errors in parentheses. For cases where χ_i is set-identified, respondents are excluded if the parameters are estimated very imprecisely (range$>$ 0.2). For all remaining set-identified parameters, the mid-point of the range is used. Observations of $\tilde{E}_{i,t}\pi_t$, $\tilde{E}_{i,t}\pi_{t+1}$, $SD_i(\pi_{t+1})$, and $SD_i(\pi_t)$ below the 1st or above the 99th percentile of that variable's distribution are also excluded as outliers, as are observations of $\tilde{\rho}_i$ outside $[-5, 5]$ (c.1% of observations).

D.4 Relationship between components of expectation formation: further details

Table 8 breaks down the different elements of the expectation law of motion according to $\tilde{\rho}_i$, divided into five categories; $\tilde{\rho}_i < 0$ $\tilde{\rho}_i = 0$, $\tilde{\rho}_i \in (0, 1)$, $\tilde{\rho}_i = 1$, and $\tilde{\rho}_i > 1$. This follows the classification for stock return beliefs in Dominitz and Manski (2011). Table 8 includes all respondents, with the most extreme 1% of responses for each of the standard deviation variables excluded as outliers.[21]

There is a highly non-linear relationship between the standard deviation of the perception and the persistence type. In particular, those who perceive that inflation is persistent but mean reverting are the most confident in their inflation perceptions. Those who believe that $\tilde{\rho}_i > 1$ have the lowest confidence in their perceptions. This fits with the notion that those who track inflation most closely are also those who have the best knowledge of its dynamic properties. Those who believe inflation has zero persistence and those who believe it is explosive tend to also believe that the noise in the inflation process is highest.

Consistent with Result 4, those who believe that $\tilde{\rho}_i > 1$ are more likely to be hand-to-mouth than any of the other persistence types, over twice as likely if one only includes

[21] Note that the first two columns include those for whom $V_i^p > V_i^f$, whose responses are inconsistent with steady-state Kalman filtering.

those whose responses are consistent with Kalman filtering. They are also much less likely to own securities.

Table 8: Means of elements of expectation laws of motion, by persistence type

	$SD_i(\pi_t)$	$SD_i(\pi_{t+1})$	$SD_i(\varepsilon_{t+1})$	χ_i	HTM	Owns Stocks
$\tilde{\rho}_i < 0$	1.58	1.79	1.65	0.85	0.09	0.65
	(0.39)	(0.11)	(0.14)	(0.02)	(0.03)	(0.04)
$\tilde{\rho}_i = 0$	1.80	1.72	1.96	0.79	0.14	0.56
	(0.09)	(0.03)	(0.04)	(0.01)	(0.01)	(0.01)
$\tilde{\rho}_i \in (0,1)$	0.90	1.59	1.68	0.80	0.09	0.67
	(0.15)	(0.07)	(0.08)	(0.02)	(0.02)	(0.03)
$\tilde{\rho}_i = 1$	1.59	1.56	1.51	0.79	0.10	0.59
	(0.20)	(0.05)	(0.06)	(0.01)	(0.01)	(0.02)
$\tilde{\rho}_i > 1$	2.33	2.01	2.21	0.92	0.19	0.47
	(0.35)	(0.09)	(0.20)	(0.01)	(0.02)	(0.03)

Note: Bundesbank-Online-panel-Households, November 2021 wave. Standard errors in parentheses. For cases where χ_i is set-identified, respondents are excluded if the parameters are estimated very imprecisely (range>0.2). For all remaining set-identified parameters, the mid-point of the range is used. Observations of $\tilde{E}_{i,t}\pi_t, \tilde{E}_{i,t}\pi_{t+1}, SD_i(\pi_{t+1})$, and $SD_i(\pi_t)$ below the 1st or above the 99th percentile of that variable's distribution are also excluded as outliers, as are observations of $\tilde{\rho}_i$ outside $[-5,5]$ (c.1% of observations). Note that the first two columns include those for whom $V_i^p > V_i^f$, whose responses are inconsistent with steady-state Kalman filtering.

Finally, note that those who believe that $\tilde{\rho}_i > 1$ have the highest χ_i on average. However, this is partly mechanical, since if $|\tilde{\rho}_i| > 1$ then that places a lower bound on the values of χ_i that are consistent with steady-state Kalman filtering. Between the groups with $\tilde{\rho}_i \in [0,1]$, the average Kalman filter varies little.

Table 9 shows regressions of each component of the expectation laws of motion on $\tilde{\rho}_i$, split in two ways. The first panel splits respondents according to which hypothetical scenario they were shown before Question 2, to explore the role of different shock types. That is, each dependent variable is regressed on $\tilde{\rho}_i$ interacted with a categorical variable reflecting which shock scenario the respondent saw.

The second panel splits households into some of the persistence categories outlined above, specifically those who believe the price level is mean-reverting ($\tilde{\rho}_i < 0$), those who believe inflation is persistent but stationary ($\tilde{\rho}_i \in (0,1)$), and those who believe inflation is non-stationary with positive persistence ($\tilde{\rho}_i \geq 1$). The final panel shows the results of regressing each dependent variable on an indicator equal to 1 if the household does no updating of expectations at all when faced with the hypothetical shock ($\tilde{\rho}_i = 0$).

In the first panel, there are some significant differences between shock types in the relationships of $\tilde{\rho}_i$ with other elements of expectation laws of motion. However, the magnitudes are generally small. For that reason we pool households across shock types for the analysis in Section 4.2.

The differences are much larger, however, across persistence types. Panel 2 shows that within households who believe inflation is persistent and stationary, greater perceived persistence is associated with less uncertainty about current and future inflation, less perceived noise in the inflation process, and a greater implied Kalman gain. This is consistent with models of endogenous information acquisition, as with a more persistent inflation process information about the current rate of inflation is more valuable.

Table 9: Breakdown of $\tilde{\rho}_i$ relationships with other expectation law of motion components by shock type and persistence category.

	(1) $SD_i(\pi_{t+1})$	(2) $SD_i(\pi_t)$	(3) $SD_i(\varepsilon_{t+1})$	(4) χ_i
Panel A: Shock type				
Shock unspecified $\times\ \tilde{\rho}_i$	-0.00278	0.0103	-0.162*	-0.00969
	(0.105)	(0.0306)	(0.0913)	(0.0149)
Supply $\times\ \tilde{\rho}_i$	0.0956	0.00437	-0.118	0.0104
	(0.105)	(0.0246)	(0.102)	(0.00983)
Demand $\times\ \tilde{\rho}_i$	0.145	-0.0546***	0.0278	0.0418***
	(0.118)	(0.0165)	(0.116)	(0.00872)
Constant	2.077***	0.751***	2.055***	0.789***
	(0.0308)	(0.0115)	(0.0307)	(0.00526)
Panel B: Persistence type				
$\tilde{\rho}_i < 0 \times \tilde{\rho}_i$	-0.150	0.0268	0.132	-0.0390***
	(0.117)	(0.0306)	(0.123)	(0.0142)
$\tilde{\rho}_i \in (0,1) \times \tilde{\rho}_i$	-0.459***	-0.321***	-0.532***	0.0497*
	(0.161)	(0.0351)	(0.162)	(0.0301)
$\tilde{\rho}_i \geq 1 \times \tilde{\rho}_i$	0.164**	-0.00462	-0.0968	0.0246***
	(0.0795)	(0.0178)	(0.0788)	(0.00727)
Constant	2.082***	0.765***	2.087***	0.784***
	(0.0340)	(0.0130)	(0.0340)	(0.00574)
Panel C: Updating indicator				
$\tilde{\rho}_i \neq 0$	-0.106*	-0.0744***	-0.306***	0.0132
	(0.0594)	(0.0196)	(0.0585)	(0.0102)
Constant	2.129***	0.770***	2.128***	0.789***
	(0.0343)	(0.0136)	(0.0343)	(0.00596)
Observations	2317	2317	2317	2317

Note: Bundesbank-Online-panel-Households, November 2021 wave.
Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Although there are only weak relationships between $\tilde{\rho}_i$ and uncertainty over current and future inflation across the whole sample, the second panel reveals that this is driven by weak relationships among those who believe in inflation processes that are qualitatively different from the data. Among those who believe that inflation is persistent but stationary, the relationships between $\tilde{\rho}_i$ and uncertainty are very strong. Since an AR(1) process estimated on German CPI inflation over the previous 20 years implies a persistence of $\rho = 0.21$, this suggests that the group of households most aware of the time-series properties of inflation behave as predicted by models of rational inattention (e.g. Sims, 2003). However outside of this group, households behave less in line with those predictions.

D.5 Correlations of expectation components with household characteristics: further details

Table 10: Probit regressions of $\tilde{\rho}_i \neq 0$ on household characteristics, split by shock type.

	(1)	(2)	(3)	(4)	(5)
	$\tilde{\rho}_i \neq 0$	$\tilde{\rho}_i \neq 0$	$\tilde{\rho}_i \neq 0$	$\tilde{\rho}_i \neq 0$	$\tilde{\rho}_i \neq 0$
Hand-to-mouth	-0.0856	-0.1078	-0.1257	-0.3049**	0.1170
	(0.1079)	(0.0761)	(0.1346)	(0.1353)	(0.1349)
Liquid wealth	0.0001	0.0000	0.0001	0.0001	-0.0000
	(0.0002)	(0.0002)	(0.0003)	(0.0003)	(0.0003)
Illiquid wealth	-0.0000	0.0000	0.0001	0.0000	-0.0000
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)
Other wealth	0.0007	0.0002	0.0002	-0.0002	0.0010
	(0.0006)	(0.0005)	(0.0008)	(0.0009)	(0.0008)
Debt	0.0004	0.0000	0.0001	-0.0007	0.0006
	(0.0003)	(0.0002)	(0.0004)	(0.0005)	(0.0004)
log(income)	-0.0624	-0.0075	-0.0890	0.0669	-0.0061
	(0.0773)	(0.0574)	(0.1036)	(0.1057)	(0.0992)
HH Controls	Yes	Yes	Yes	Yes	Yes
Shock type	All	All	Unspecified	Supply	Demand
Observations	1899	3194	1053	1051	1068
Pseudo-R^2	0.0285	0.0177	0.0329	0.0405	0.0397

Note: Bundesbank-Online-panel-Households, November 2021 wave. The units of the wealth and debt variables are €1000s. The household controls are age (in years up to a top bin of ≥ 80, coded as 80), age^2, gender, region (north/south/east/west), education, occupation category, and employment status (all categorical, for details see the full questionnaire at https://www.bundesbank.de/en/bundesbank/research /survey-on-consumer-expectations/questionnaires-850746). All controls except age and age^2 are treated as categorical. Robust standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 10 column 1 shows the results of a probit regression of of an indicator variable

w_i on household characteristics, where $w_i = 1$ if $\tilde{\rho}_i \neq 0$, and $= 0$ otherwise. These results therefore give the estimated relationship between household characteristics and the probability of adjusting expectations in light of hypothetical shocks. None of the characteristics are significantly related to this selection, and the magnitudes are small: the average marginal effect of being hand-to-mouth on the probability of $\tilde{\rho}_i \neq 0$ is less than 3 percentage points.

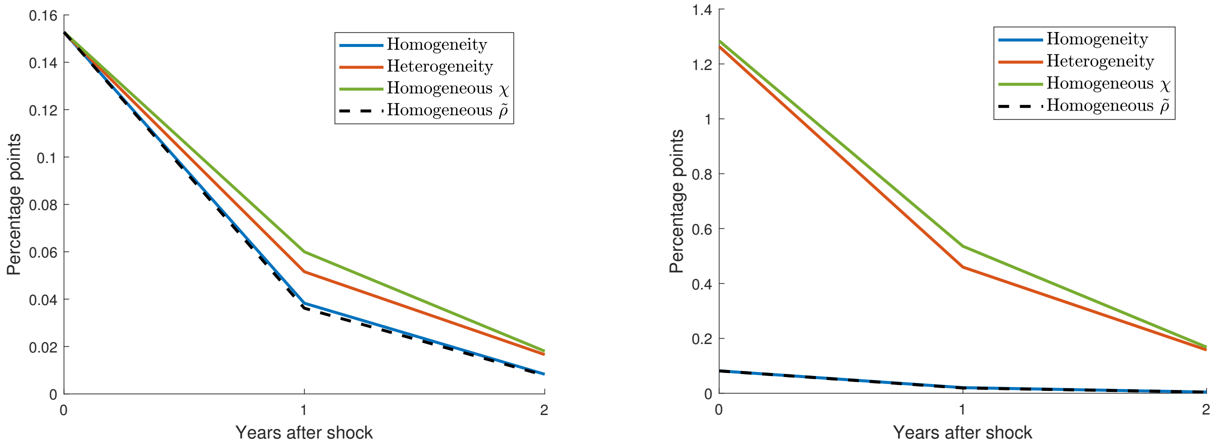
This result is robust to extending the sample to include those for whom χ_i or $\tilde{\sigma}^2_{\varepsilon i}$ cannot be inferred precisely: as those variables are not used in the calculation of $\tilde{\rho}_i$, the remaining columns of Table 10 repeat the exercise for the full sample, and then split by the shock scenario seen by the household. The associations between $\tilde{\rho}_i \neq 0$ and wealth/income is not significantly different from zero throughout, except for the supply shock, for which hand-to-mouth households are somewhat less likely than others to adjust their expectations.

E Further impulse response exercises

In Figure 10b, we break down the amplification from heterogeneous expectation formation into its components. The impulse response with heterogeneity in $\tilde{\rho}_i$ only, but homogeneous χ_i, is close to that with full heterogeneity. This is therefore the main driver of the amplification we find. Note however that the difference between the IRFs with full heterogeneity and with homogeneous χ_i is small relative to the response with full heterogeneity, it remains large relative to the consumption responses with homogeneous expectation formation, and with homogeneous $\tilde{\rho}_i$. The covariance between $\tilde{\rho}_i$ and χ_i, though small in the data, does still play a non-trivial role in aggregate consumption dynamics.

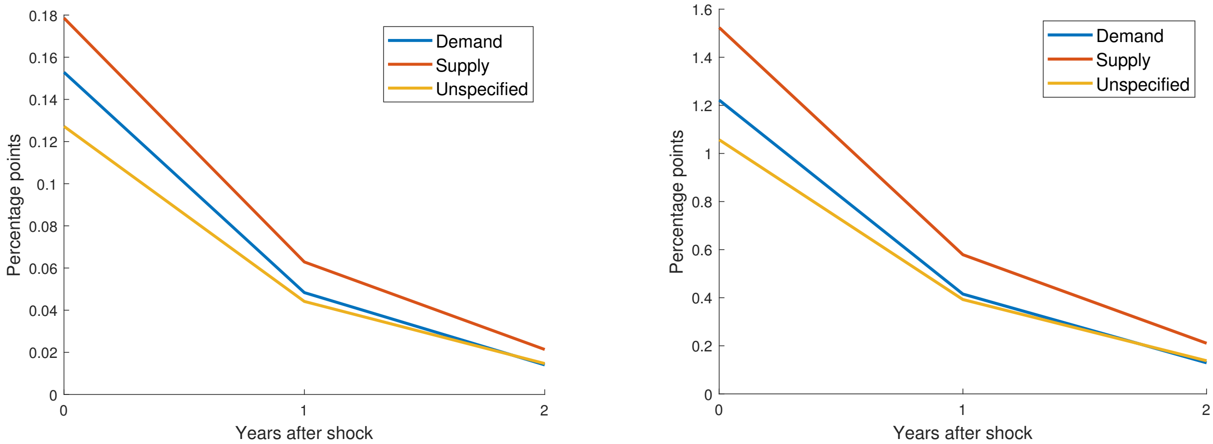
Using the survey responses to the different hypothetical scenarios in Question 2, we can further compare the effects of heterogeneous expectation laws of motion for different types of shock. We find somewhat greater amplification and persistence in consumption responses to supply shocks than other types of shock. A comparison of the IRFs between the three cases is shown in Figures 11a and 11b. This result is consistent with the higher average perceived persistence of supply shocks discussed in Section 4.

As in Section 4, we also repeat these exercises with the distributions of subjective models after excluding those whose answers are rounded to a multiple of 0.5. The model with heterogeneity does deliver smaller initial consumption responses in this case, but it is still 4.4× larger than under the homogeneity case calibrated using the average $\tilde{\rho}_i$ and χ_i across all respondents. This rises to 5.5× if one compares to a representative agent model calibrated using the average $\tilde{\rho}_i$ and χ_i for the population of non-rounders (as they have slightly smaller perceived persistence on average). As such, the result



(a) Expectations **(b)** Consumption

Figure 10: Implied IRF of aggregate expectations and aggregate consumption. The homogeneity and heterogeneity cases are as described in Section 5.2. The remaining cases set χ_i and $\tilde{\rho}_i$ respectively to their average values for all households. Source: Bundesbank-Online-Panel-Households, November 2021 wave.



(a) Expectations **(b)** Consumption

Figure 11: Implied IRFs of one-period ahead inflation expectations and consumption by shock. Source: Bundesbank-Online-Panel-Households, November 2021 wave.

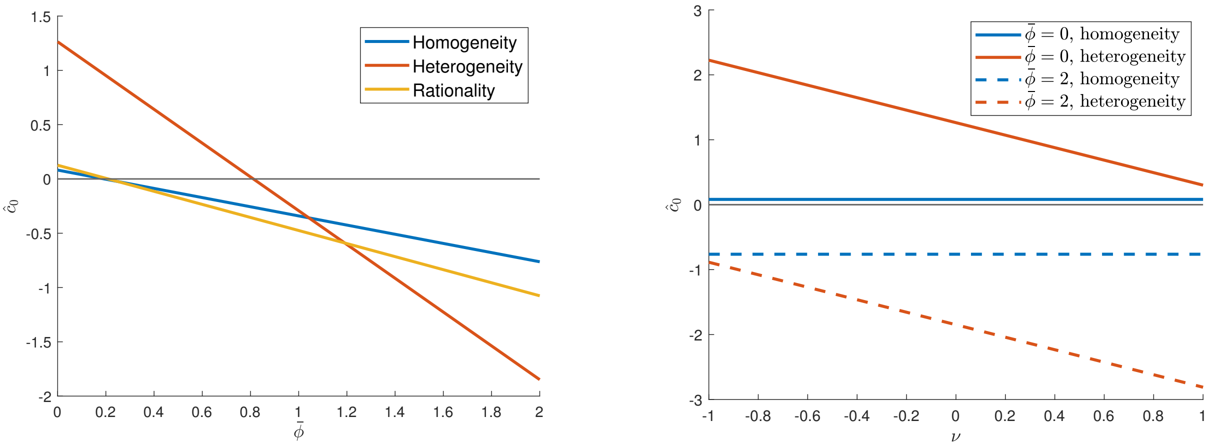
that heterogeneity generates very significant amplification of the transmission of inflation shocks to consumption still holds.

Finally, we allow for households to expect nominal interest rates to respond to inflation. Specifically, we assume that:

$$\tilde{E}_{i,t} i_{t+h} = \tilde{\phi}_i \tilde{E}_{i,t} \pi_{t+h} \tag{55}$$

Households are assumed to not observe i_t precisely when they choose $c_{i,t}$, consistent with them not observing π_t. They therefore infer i_t from their perceived current inflation, just as they do for expectations of future periods.

Figure 12 shows the initial consumption response $\hat{c}_0$ under homogeneity, heterogeneity, and rationality for a range of assumptions about $\tilde{\phi}_i$. In panel (a), we assume that all households share a common $\tilde{\phi}_i = \bar{\phi}$. When the Taylor principle is expected to be satisfied, higher inflation leads households to expect higher real interest rates, and so to reduce consumption. Heterogeneity amplifies this fall in consumption, just as it amplifies the consumption increase in the baseline case ($\bar{\phi} = 0$) studied in Section 5.2. Indeed, heterogeneity provides substantial amplification of aggregate consumption for all perceived interest rate rules aside from a small region around $\bar{\phi} = 1$, in which inflation shocks are only expected to have very small effects on real interest rates. Heterogeneity also increases the sensitivity of aggregate consumption to the perceived interest rate rule. In panel (b), we show that our results also remain robust even if $\tilde{\phi}_i$ is allowed to covary with $\tilde{\rho}_i$. Such covariances do make some quantitative difference to the results, so future research could consider ways to measure these relationships.



(a) Varying $\bar{\phi}$ **(b)** Varying ν

Figure 12: Implied $\hat{c}_0$ under different assumptions on $\tilde{\phi}_i$. In panel (a) all households form interest rate expectations using the same perceived Taylor Rule parameter $\bar{\phi}$. In panel (b) this parameter varies across households according to $\tilde{\phi}_i = \bar{\phi} + \nu(\tilde{\rho}_i - E[\tilde{\rho}_i])$. Source: Bundesbank-Online-Panel-Households, November 2021 wave.